A Clustering Approach for Collaborative Filtering under the Belief Function Framework

Raoua Abdelkhalek, Imen Boukhris and Zied Elouedi

Lugano, July 12th, 2017
1. Introduction

2. Belief Function Theory

3. Collaborative Filtering Recommender

4. Evidential Clustering Collaborative Filtering

5. Experimental Study

6. Conclusion and Future Works
1 Introduction

2 Belief Function Theory

3 Collaborative Filtering Recommender

4 Evidential Clustering Collaborative Filtering

5 Experimental Study

6 Conclusion and Future Works
What is the suitable country to visit?
Which book should I buy for my next vacation?
Which movie should I watch?
etc.
Context
How much information can we handle?

- A plethora of information
- Confusion.
- Wasting time.
- Reaching unsatisfiable options.
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A plethora of information

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⇒ Recommender systems provide recommendations based on users’ preferences.
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The collaborative filtering (CF) is the most widely used recommendation approach. The collaboration between users to filter out relevant items.
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The collaboration between users to filter out relevant items.
Collaborative filtering approach

User-based
- Predicts the active user’s preferences based on past ratings from users similar to him.

Item-based
- Computes how similar a set of items the active user has rated, to the target item.
Collaborative filtering approach

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Collaborative filtering approach

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<tbody>
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Collaborative filtering approach

**User-based**
- Predicts the active user’s preferences based on past ratings from users similar to him.

**Item-based**
- Computes how similar a set of items the active user has rated, to the target item.

**Items:** The products on which the recommender system aims to predict the users’ preferences.
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Item-based CF approach

Item-Based CF

Selecting similar items in the system to predict the user’s preferences.
Item-based CF approach

Item-Based CF
Selecting similar items in the system to predict the user’s preferences.
### Item-based CF approach

Example

<table>
<thead>
<tr>
<th>Similarity</th>
<th>Movie 1</th>
<th>Movie 2</th>
<th>Movie 3</th>
<th>Movie 4</th>
<th>Movie 5</th>
<th>Movie 6</th>
<th>Movie 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>User 1</td>
<td>2</td>
<td>4</td>
<td>?</td>
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**Item-based CF approach**

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Item-based CF approach

Example

MovieLens recommends these movies

**top picks**

<table>
<thead>
<tr>
<th>Movie</th>
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</tr>
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<tbody>
<tr>
<td>Inception</td>
<td>2010</td>
<td>PG-13</td>
</tr>
<tr>
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<td>2011</td>
<td>PG-13</td>
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Motivation

Limitations

- CF needs to search the whole user-item space to compute items similarities.
- This computation leads to poor scalability performance.
- Clustering items to reduce the consuming time.
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  $\Rightarrow$ Clustering items to reduce the consuming time.
Motivation

Problem statement

- Does not take into account the uncertainty involved during the clusters assignments.
- An item may potentially belong to more than only one cluster.

⇒ Soft clustering
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Objectives

1. Dealing with the **uncertain** aspect of the items’ clustering.
2. Improving the **scalability** of the CF approach under uncertainty.
   ⇒ Maintaining a good recommendation performance.

Goal

A Clustering Approach for Collaborative Filtering under the Belief Function Framework
Objectives

1. Dealing with the uncertain aspect of the items’ clustering.

2. Improving the scalability of the CF approach under uncertainty.

=> Maintaining a good recommendation performance.

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Goal

A Clustering Approach for Collaborative Filtering under the Belief Function Framework
Definition

- A flexible and rich framework for dealing with imperfect information.

Frame of discernment: $\Theta$

$\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$

$2^\Theta = \{A : A \subseteq \Theta\}$

Basic belief assignment: $bba$

$m : 2^\Theta \rightarrow [0, 1]$

$\sum_{A \subseteq \Theta} m(A) = 1$
Belief function theory

Basic Concepts

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The combination rules combine the bba’s induced from independent information sources into a unique one.

Dempster’s rule of combination

\[(m_1 \oplus m_2)(A) = k \sum_{B,C \subseteq \Theta: B \cap C = A} m_1(B) \cdot m_2(C)\]

\[k^{-1} = 1 - \sum_{B,C \subseteq \Theta: B \cap C = A} m_1(B) \cdot m_2(C) \text{ and } (m_1 \oplus m_2)(\emptyset) = 0\]
Belief function theory
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\]
Pignistic probability

\[
\text{BetP}(A) = \sum_{B \subseteq \Theta} \frac{|A \cap B|}{|B|} \frac{m(B)}{1 - m(\emptyset)} \quad \text{for all } A \in \Theta
\]
Examples

- Belief K-modes: Dealing with uncertainty in the attribute values.
- Evidential C-means: Handling uncertainty for objects’ assignment.
- ...
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Belief function theory
Clustering techniques
Examples

- **Belief K-modes**: Dealing with uncertainty in the attribute values.
- **Evidential C-means**: Handling uncertainty for objects’ assignment.
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Evidential C-means

- Allows a credal partition of the objects.

**Principle**

- Determining the mass $m_i$ representing partial knowledge regarding the cluster membership to any subset of $\Theta$.
- $\Theta = \{\omega_1, \omega_2, \ldots, \omega_n\}$.
- $n$ is the number of clusters.
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Evidential C-means

ECM

Principle

- Every partition is represented by a prototype \( v_k \in \mathbb{R}^p \).
- Each subset \( A_j \) of \( \Theta \) is represented by the barycenter \( v_j \) of the centers \( v_k \).
Objective Criterion

The credal partition is determined by minimizing the following objective function:

\[
J_{ECM} = \sum_{i=1}^{n} \left( \sum_{\{j/A_j \neq \emptyset, A_j \subseteq \Theta\}} |A_j^\alpha| m_{ij}^\beta d_{ij}^2 + \sum_{i=1}^{n} \delta^2 m_{i\emptyset}^\beta \right)
\]

- $\alpha \geq 0$ is a weighting exponent for cardinality.
- $\beta > 1$ is a weighting exponent controlling the hardness of the partition.
- $\delta$ represents the distance between all instances and the empty set.
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Basic concepts

- **Target item**: The current item for which we would like to predict users’ preferences
- **Active user**: The user for whom the task is to find items’ suggestions.
- **Rating** $r_{u,i}$: The preference expressed by the user $u$ for the item $i$ in the system.
- **User-item matrix**: The set of all rating triples (User, Item, Rating).
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Collaborative Filtering

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<th>i_3</th>
<th>i_4</th>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>r_{a,n}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>u_m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>r_{m,1}</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>r_{m,2}</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

User-item matrix

Target item

Active user

Prediction

Recommendation
Collaborative Filtering
Outline

1. Introduction
2. Belief Function Theory
3. Collaborative Filtering Recommender
4. Evidential Clustering Collaborative Filtering
5. Experimental Study
6. Conclusion and Future Works
Evidential Clustering CF

Step 1
Items Clustering

Step 2
Clusters Selection

Step 3
Ratings Prediction
Evidential Clustering CF

Step 1
- Items Clustering

Step 2
- Clusters Selection

Step 3
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Step 1: Items Clustering

**Principle**

1. Exploiting the user-item matrix and randomly initializing the cluster centers.

2. Computing the euclidean distance between the items and the non empty subsets of $\Theta$.

3. Allocating for each item in the matrix a mass of belief to any subsets of the $\Theta$.

$\Rightarrow$ Credal partition.
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Step 1: Items Clustering

Example

User-item matrix

<table>
<thead>
<tr>
<th></th>
<th>Movie₁</th>
<th>Movie₂</th>
<th>Movie₃</th>
<th>Movie₄</th>
<th>Movie₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>User₁</td>
<td>3</td>
<td>?</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>User₂</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>User₃</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>User₄</td>
<td>?</td>
<td>1</td>
<td>5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>User₅</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

The clustering process consists of providing a credal partition for the 5 movies.
Step 1: Items Clustering

Example

User-item matrix

<table>
<thead>
<tr>
<th></th>
<th>Movie_1</th>
<th>Movie_2</th>
<th>Movie_3</th>
<th>Movie_4</th>
<th>Movie_5</th>
</tr>
</thead>
<tbody>
<tr>
<td>User_1</td>
<td>3</td>
<td>?</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>User_2</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>User_3</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>User_4</td>
<td>?</td>
<td>1</td>
<td>5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>User_5</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

- The clustering process consists of providing a credal partition for the 5 movies.
Step 1: Items Clustering

Example

The credal partition corresponding to the five movies (c=3)

<table>
<thead>
<tr>
<th></th>
<th>$\emptyset$</th>
<th>${C_1}$</th>
<th>${C_2}$</th>
<th>${C_1, C_2}$</th>
<th>${C_3}$</th>
<th>${C_1, C_3}$</th>
<th>${C_2, C_3}$</th>
<th>$\Theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>0.002</td>
<td>0.968</td>
<td>0.009</td>
<td>0.007</td>
<td>0.004</td>
<td>0.004</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.046</td>
<td>0.294</td>
<td>0.271</td>
<td>0.110</td>
<td>0.113</td>
<td>0.073</td>
<td>0.051</td>
<td>0.038</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0.005</td>
<td>0.001</td>
<td>0.001</td>
<td>0.004</td>
<td>0.993</td>
<td>0.009</td>
<td>0.001</td>
<td>0.004</td>
</tr>
<tr>
<td>$M_4$</td>
<td>0.006</td>
<td>0.021</td>
<td>0.885</td>
<td>0.017</td>
<td>0.024</td>
<td>0.010</td>
<td>0.024</td>
<td>0.009</td>
</tr>
<tr>
<td>$M_5$</td>
<td>0.036</td>
<td>0.148</td>
<td>0.493</td>
<td>0.090</td>
<td>0.094</td>
<td>0.047</td>
<td>0.055</td>
<td>0.032</td>
</tr>
</tbody>
</table>
Step 2: Clusters Selection

**Principle**

- Computing the pignistic probability $BetPi$ induced by each $bba$.
- Assigning each item to the cluster with the highest pignistic probability.
Step 2: Clusters Selection

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- Assigning each item to the cluster with the highest pignistic probability.
Step 2: Clusters Selection

Example

The pignistic probabilities corresponding to the five movies

<table>
<thead>
<tr>
<th>Movies</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>Selected cluster</th>
</tr>
</thead>
<tbody>
<tr>
<td>Movie$_1$</td>
<td>0.9773</td>
<td>0.0144</td>
<td>0.0083</td>
<td>?</td>
</tr>
<tr>
<td>Movie$_2$</td>
<td>0.4188</td>
<td>0.3833</td>
<td>0.1979</td>
<td>?</td>
</tr>
<tr>
<td>Movie$_3$</td>
<td>0.0017</td>
<td>0.0029</td>
<td>0.9953</td>
<td>?</td>
</tr>
<tr>
<td>Movie$_4$</td>
<td>0.0387</td>
<td>0.9155</td>
<td>0.0458</td>
<td>?</td>
</tr>
<tr>
<td>Movie$_5$</td>
<td>0.2374</td>
<td>0.5992</td>
<td>0.1633</td>
<td>?</td>
</tr>
</tbody>
</table>

Making a final decision about the cluster of each movie.
Step 2: Clusters Selection

Example

The pignistic probabilities corresponding to the five movies

<table>
<thead>
<tr>
<th>Movies</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>Selected cluster</th>
</tr>
</thead>
<tbody>
<tr>
<td>Movie₁</td>
<td>0.9773</td>
<td>0.0144</td>
<td>0.0083</td>
<td>C₁</td>
</tr>
<tr>
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<td>0.3833</td>
<td>0.1979</td>
<td>C₁</td>
</tr>
<tr>
<td>Movie₃</td>
<td>0.0017</td>
<td>0.0029</td>
<td>0.9953</td>
<td>C₃</td>
</tr>
<tr>
<td>Movie₄</td>
<td>0.0387</td>
<td>0.9155</td>
<td>0.0458</td>
<td>C₂</td>
</tr>
<tr>
<td>Movie₅</td>
<td>0.2374</td>
<td>0.5992</td>
<td>0.1633</td>
<td>C₂</td>
</tr>
</tbody>
</table>

- Selecting the corresponding cluster having the highest value.
Step 3: Ratings Prediction

Example

**Principle**

- Extracting the items belonging to the same cluster as the target item.
- Computing the average of the ratings corresponding to the same clusters members.
Step 3: Ratings Prediction

Example

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- Extracting the items belonging to the same cluster as the target item.
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Rating Computation

- The rating prediction is performed as follows:

\[
\hat{R}_{u,i} = \frac{\sum_{j \in C_i(u)} R_{uj}}{|C_i(u)|}
\]

- \(C_i(u)\) is the set of items to the cluster of the item \(i\) and rated by the user \(u\).
- \(R_{uj}\) is the rating given by user \(u\) to item \(j\).
- \(|C_i(u)|\) is the number of items rated by user \(u\) in cluster \(C_i\).
Step 3: Ratings Prediction

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### User-item matrix

<table>
<thead>
<tr>
<th></th>
<th>Movie(_1)</th>
<th>Movie(_2)</th>
<th>Movie(_3)</th>
<th>Movie(_4)</th>
<th>Movie(_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>User(_1)</td>
<td>3</td>
<td>?</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>User(_2)</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>User(_3)</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>User(_4)</td>
<td>?</td>
<td>1</td>
<td>5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>User(_5)</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Clusters</td>
<td>C(_1)</td>
<td>C(_1)</td>
<td>C(_3)</td>
<td>C(_2)</td>
<td>C(_2)</td>
</tr>
</tbody>
</table>

- The rating \(\hat{R}_{1,2}\) given by User\(_1\) to Movie\(_2\) ?
Step 1: Items Clustering

Example

User-item matrix

<table>
<thead>
<tr>
<th></th>
<th>Movie₁</th>
<th>Movie₂</th>
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<tr>
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<td>3</td>
<td>?</td>
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<td>4</td>
<td>4</td>
<td>2</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>User₃</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
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<td>?</td>
<td>1</td>
<td>5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>User₅</td>
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<td>2</td>
<td>0</td>
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<tr>
<td>Clusters</td>
<td>C₁</td>
<td>C₁</td>
<td>C₃</td>
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<td>C₂</td>
</tr>
</tbody>
</table>

- The rating \( \hat{R}_{1,2} \) given by User₁ to Movie₂?
- Movie₂ and Movie₁ ∈ C₁.
- \( \hat{R}_{1,2} = \frac{3}{1} = 3 \).
MoviesLens

- 943 users
- 1682 movies
- 100,000 ratings
- Ratings scale: [1,5]
Evaluation metrics

Mean Absolute Error (MAE)

- Evaluating the prediction accuracy.

\[
MAE = \frac{\sum_{u,i} |\hat{R}_{u,i} - R_{u,i}|}{||\hat{R}_{u,i}||}
\]

- \( R_{u,i} \): Real rating for the user \( u \) on the item \( i \)
- \( \hat{R}_{u,i} \): Predicted rating
- \( ||\hat{R}_{u,i}|| \): Total number of the predicted ratings.

\( \Rightarrow \) Lower values of MAE = Better prediction accuracy
Evaluating the prediction accuracy.

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⇒ Lower values of \(MAE\) = Better prediction accuracy
### Precision

- Evaluating the quality of recommendations.

\[
\text{Precision} = \frac{IR}{IR + UR}
\]

- **IR**: Interesting item has been correctly recommended
- **UR**: Uninteresting item has been incorrectly recommended

⇒ Higher precision values = Better performance
Evaluation metrics

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Evaluation metrics

Scalability Performance
- The ability of the recommendation approach to be run quickly.
Comparative protocol

Proposed Approach
- Evidential clustering item-based CF (EC-IBCF)

VS

Traditional Approach
- Evidential item-based CF (EV-IBCF)
Comparative protocol

Proposed Approach
- Evidential clustering item-based CF (EC-IBCF)

VS

Traditional Approach
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Traditional Approach
- Evidential item-based CF (EV-IBCF)
Experimental results

- Performance in terms of prediction and recommendation

<table>
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<tr>
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<tbody>
<tr>
<td>Mean_MAE</td>
<td>EV-IBCF 0.809</td>
<td>EC-IBCE 0.793</td>
</tr>
<tr>
<td>Mean_Precision</td>
<td>0.733</td>
<td>0.75</td>
</tr>
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EC-IBCE has the lowest error values in terms of Mean_MAE.
EC-IBCE achieves better results in terms of Mean_Precision.
Experimental results

- Performance in terms of prediction and recommendation

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⇒ EC-IBCE has the lowest error values in terms of Mean_MAE.
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Experimental results

- Scalability Performance

⇒ The execution time of the clustering CF approach is substantially lower than the basic evidential CF.
Outline

1. Introduction
2. Belief Function Theory
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4. Evidential Clustering Collaborative Filtering
5. Experimental Study
6. Conclusion and Future Works
Conclusion and future works

Conclusion

- Maintaining a good scalability and recommendation performance.

Future works

- Relying on the different bba’s corresponding to the different clusters rather that the most significant one.
Conclusion

- Maintaining a good scalability and recommendation performance.

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Thank you