Differences of Opinion

Dionissi Aliprantis
Cleveland Fed

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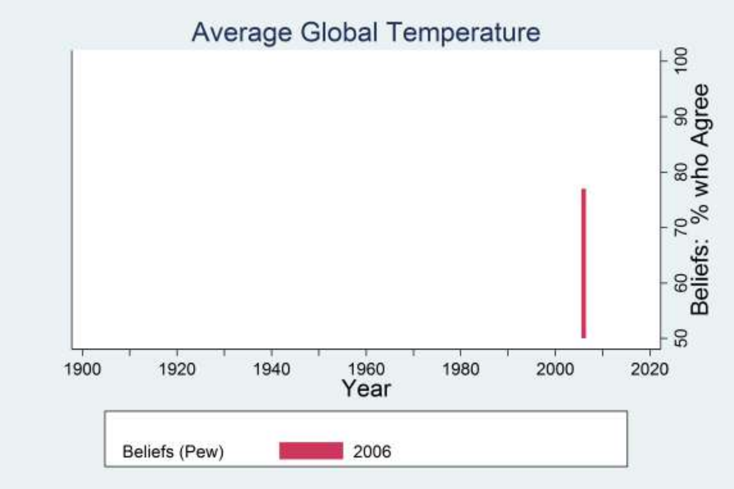

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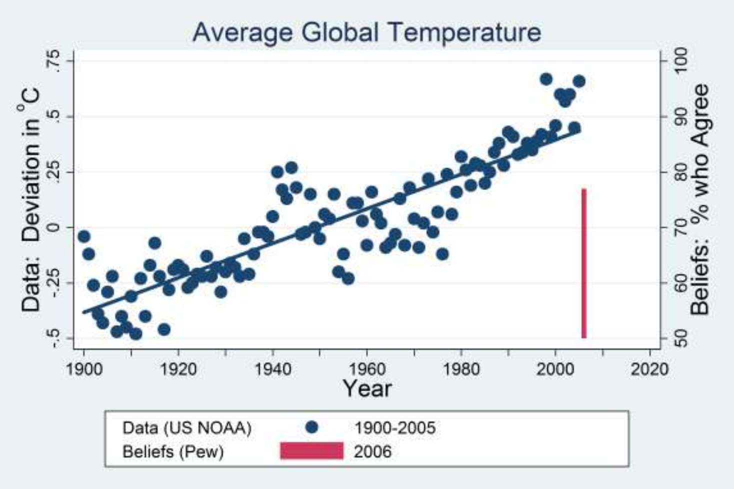

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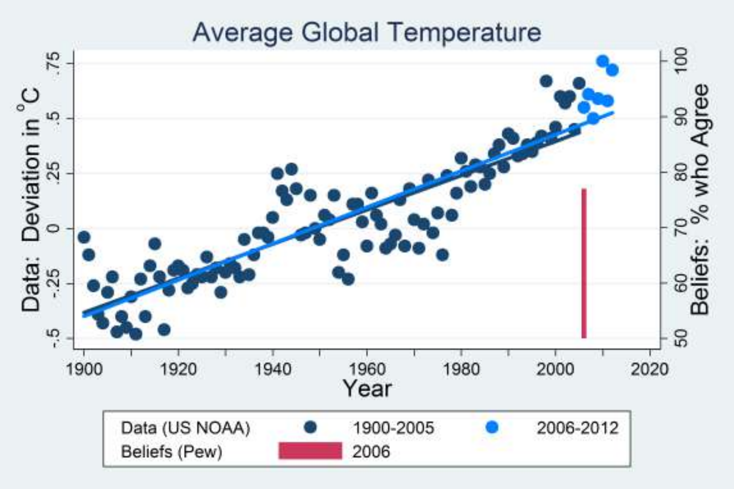

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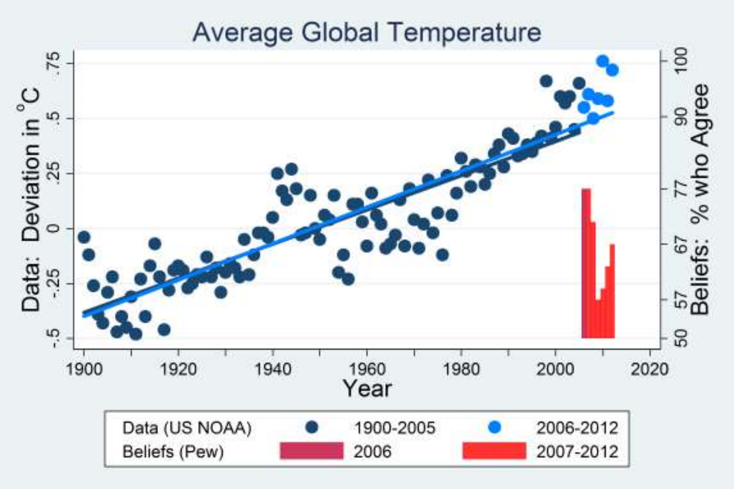

Inference about a Binary State

Proposition p^k: A statement with truth value $T(p^k) \in \{0, 1\}$

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The earth is warming.

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Non-causal examples:

The earth is warming.
The economy is in a recession.

Causal examples:

Neighborhoods affect employment.
Extending unemployment benefits increases unemployment.
Increasing the minimum wage increases unemployment.

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Directly-Observed Data + Model $\Longrightarrow$ Point-Valued Signal

$$\sigma_{it}^{k*} = \varphi_i^k(W_{it}^*) \qquad = Pr[T(p^k) = 1 | W_{it}^*] \in [0, 1]$$

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Weak Law of Large Numbers:

$$\lambda_{it+1}^{k*} = \frac{1}{t}\sum_{n=1}^{t} \sigma_{in}^{k*}$$

$$= (1-\delta_t)\lambda_{it}^{k*} + \delta_t \sigma_{it}^{k*} \quad \text{where} \quad \delta_t = 1/t$$

For any $\epsilon > 0$ $\quad \lim_{t\to\infty} \Pr\left(|\lambda_{it}^{k*} - E[\sigma_i^{k*}]| > \epsilon\right) = 0$

Partial Identification

Ex: W_{it} is state GDP along with random changes to state spending

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Point-Valued Signal σ_{it}^k under least credible assumptions

Set-Valued Signal $[\underline{\sigma}_{it}^{k*}, \overline{\sigma}_{it}^{k*}]$ under most credible assumptions

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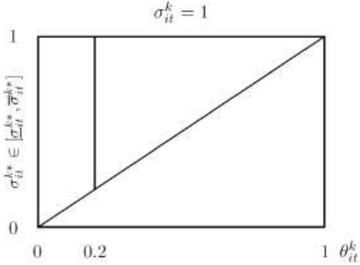

Decision Making under Ambiguity

Decision Rules

1) Choose one belief from the set of possible beliefs
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1A) Use a rule based on utility

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Gilboa and Schmeidler (1989)

ii) Min the max regret from not knowing true state

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1B) Infer missing data using info from social network

This Paper

Replicating Direct Observation

<u>The Agent's Problem:</u> Construct $\widehat{\sigma}_{it}^{k} = \sigma_{it}^{k*}$

$$\widehat{\sigma}_{it}^{k} = \underbrace{\theta_{it}^{k}}_{\substack{\text{share of signal} \\ \text{directly-observed}}} \sigma_{it}^{k} + \underbrace{(1-\theta_{it}^{k})}_{\substack{\text{share of signal} \\ \text{socially-observed}}} \sigma_{Jt}^{k}$$

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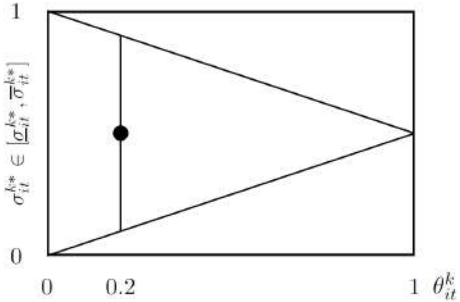

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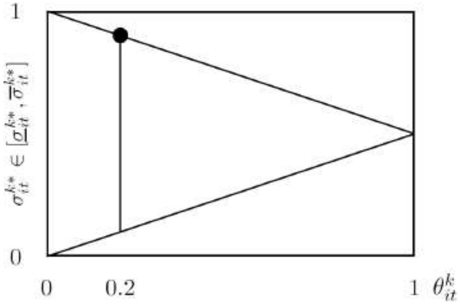

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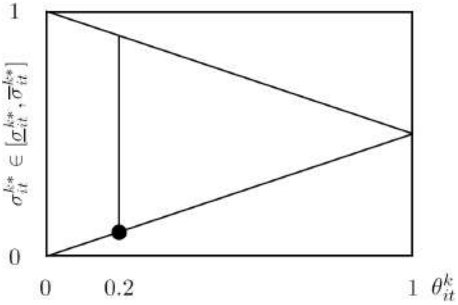

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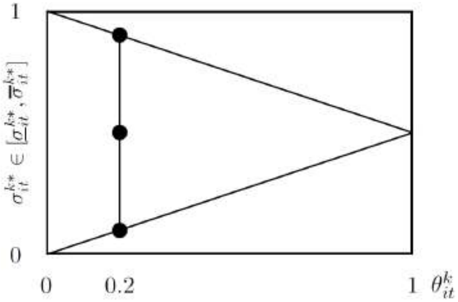

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$\Rightarrow$ Given W_{it}, $\{\sigma_{jt}^{k}\}_{1,1}^{K,J}$, $\{\lambda_{jt}^{k}\}_{1,1}^{K,J}$ choose σ_{Jt}^{k} so that $\widehat{\sigma}_{it}^{k} = \sigma_{it}^{k*}$

Problem of Inference: Agent does not observe W_{jt} or φ_{j}^{k}

Linear Opinion Pooling $\implies$ DeGroot Updating

Linear Opinion Pooling

$$\widehat{\sigma}_{it}^{k} = \theta_i^k \sigma_{it}^k + (1 - \theta_i^k)\sigma_{Jt}^k$$

$$\sigma_{Jt}^k = \sum_{j \in \mathcal{J}^k} w_{jt}^k \sigma_{jt}^k \qquad \text{with} \quad w_{jt}^k \geq 0 \ \ \forall \ \ j \in \mathcal{J}^k, \quad \sum_{j \in \mathcal{J}^k} w_{jt}^k = 1$$

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DeGroot Updating if:

1. Entire network directly observes data once at $t = 1$
2. Each agent sets $\lambda_{i1}^k = \sigma_{i1}^k$
3. Signals = beliefs for $j \geq 2$ ($\sigma_{it}^k = \lambda_{it}^k$ and $\sigma_{jt}^k = \lambda_{jt}^k$)

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Solves the agent's problem if:

$$\mathbb{E}[\sigma_{it}^{k*}] = \mathbb{E}[\sigma_{jt}^k] \quad \forall j \in \mathcal{J}^k$$

What if sender *j*'s signals are "biased"?

Linear Opinion Pooling with Interpreted Signals

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Solves the agent's problem if:

She has $f^k \in \mathcal{F}$ such that $\mathbb{E}[\sigma_{it}^{k*}] = \mathbb{E}[s_{jt}^k] \;\; \forall \;\; j \in \mathcal{J}^k$

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The Fundamental Problem of Inference

σ_{it}^{k*} is never observed

Finding $f^k \in \mathcal{F}$ is an Ill-Posed Problem

One "Reasonable" Heuristic

$F(\sigma_{it}^{k*} - \sigma_{jt}^{k})$ Is Driven by:

I (Random Sampling Error): $\Gamma_i^* = \Gamma_j$

II (Biased Sampling Process): $\Gamma_i^* \neq \Gamma_j$

III (Different Models): $\varphi_i^k \neq \varphi_j^k$

IV (Social Influence): j's model of social learning/network/etc.

V (Strategic Reporting)

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If (I)-(III) drive $F(\sigma_{it}^{k*} - \sigma_{jt}^{k}) \quad \Rightarrow \quad \mathbb{E}\left[\lambda_{it}^{k*} - \lambda_{jt}^{k}\right] = \mathbb{E}\left[\sigma_{it}^{k*} - \sigma_{jt}^{k}\right] \quad \Rightarrow$

$$s_{jt}^k = \sigma_{jt}^k + \left(\lambda_{it}^{k*} - \lambda_{jt}^k\right) \qquad \text{(H1)}$$

solves the agent's problem

Assessing Heuristic Credibility

Linear Opinion Pooling

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Relative Entropy of $\lambda_{it}^k - \lambda_{jt}^k$ over Propositions

Use to assign credibility weights w_{jt}^k to signals interpreted using $\widehat{\text{H1}}$

Idea: Give more weight to senders that are better understood

Sethi and Yildiz (2016)

Experiment

Network: J+1=300 learning about $K = 30$ propositions

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Network: J+1=300 learning about $K = 30$ propositions

Data: Quality $\theta_{it}^{k} = 0.1 \;\; \forall \;\; k, i, t$ with signals: $\sigma_{it}^{k} = 0.5 \;\; \forall \;\; k, i, t$

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Inference via $\widehat{\text{H1}}$: $\hat{\sigma}_{it}^k = \theta_{it}^k \sigma_{it}^k + (1 - \theta_{it}^k) s_{Jt}^k$ where $s_{jt}^k = \sigma_{jt}^k + \left(\lambda_{it}^k - \lambda_{jt}^k\right)$ and

$$s_{Jt}^k = \sum_{j=1}^{J^k} w_{jt}^k s_{jt}^k \quad \text{where} \quad w_{jt}^k = \frac{\Delta_{ijt}}{\sum_{j=1}^{J^k} \Delta_{ijt}} \quad \text{and}$$

$$Q_{ijt} \equiv f_k(\lambda_{it}^k - \lambda_{jt}^k)$$
$$\Delta_{ijt} \equiv \rho(D_{KL}(Q_{ijt} : U)) = \left[\gamma_1 D_{KL}(Q_{ijt} : U)\right]^{\gamma_2} \quad \text{where} \quad (\gamma_1, \gamma_2) = (100, 8)$$

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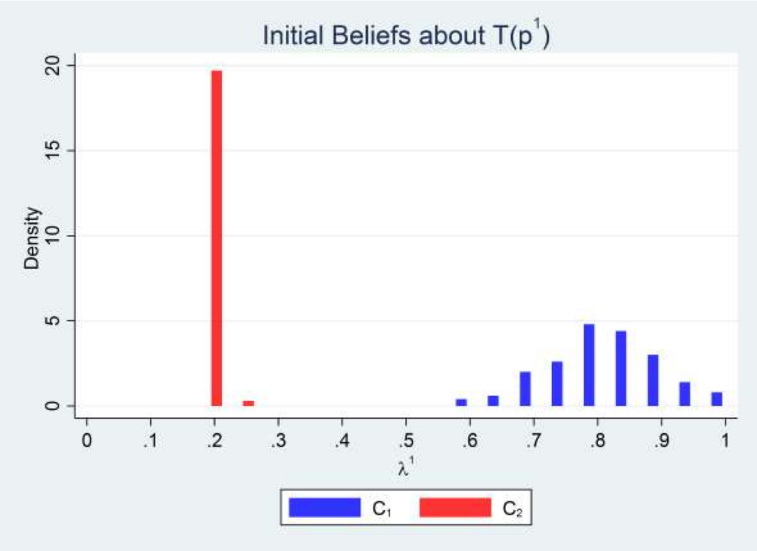
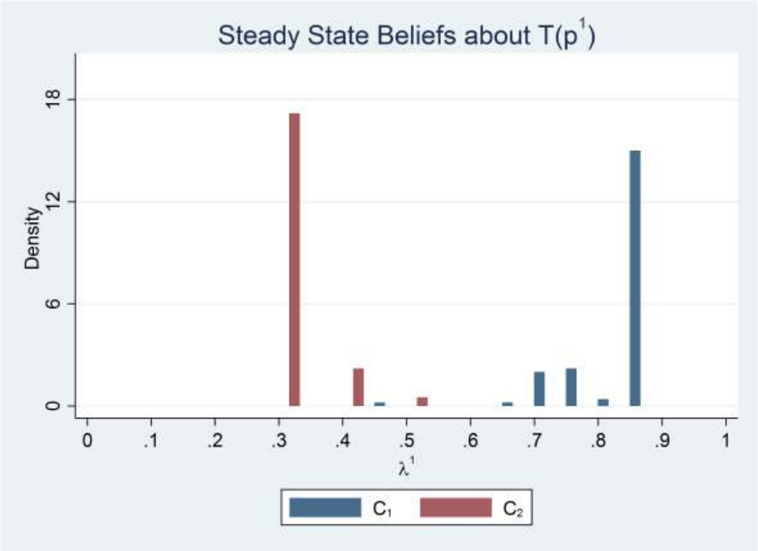
Initial Beliefs:

$$\overline{\lambda}_{i1}^k \sim \begin{cases} \mathcal{N}(0.8, 0.1) & \text{if } i \in \mathcal{C}_1 \quad \forall k = 1, \ldots, K \qquad card(\mathcal{C}_1) = 100; \\ \mathcal{N}(0.2, 0.01) & \text{if } i \in \mathcal{C}_2 \quad \forall k = 1, \ldots, K \qquad card(\mathcal{C}_2) = 200. \end{cases}$$

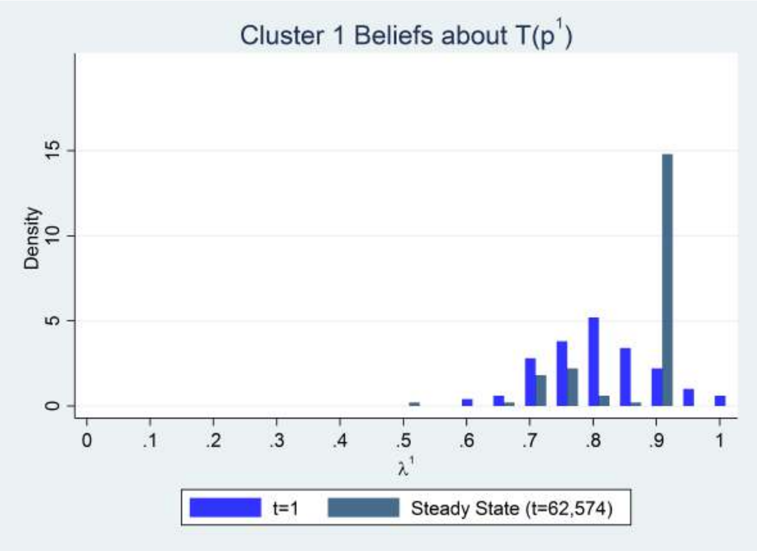
Experiment


Initial Beliefs about T(p¹)
Density
0
5
10
15
20
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
λ¹
C₁
C₂

Experiment


Steady State Beliefs about $T(p^1)$
Density
0
6
12
18
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
λ^1
C_1
C_2

Experiment


Cluster 1 Beliefs about T(p1)
Density
0
5
10
15
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
λ1
t=1
Steady State (t=62,574)

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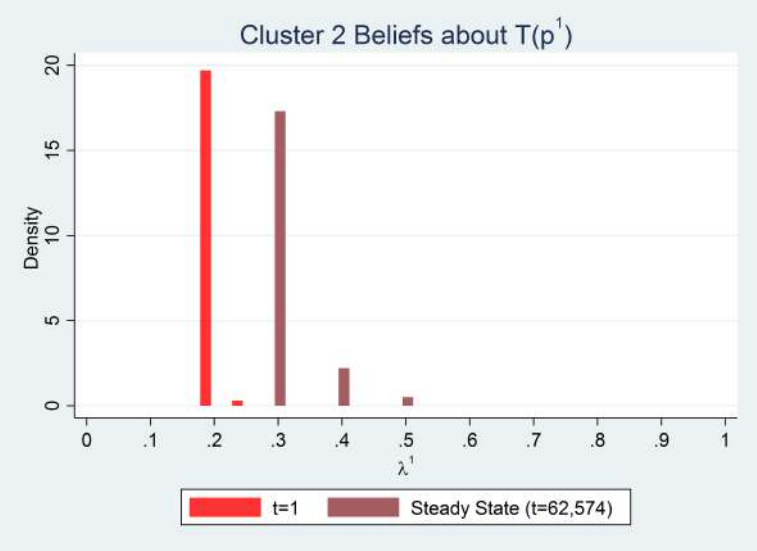
Cluster 2 Beliefs about T(p^1)
Density
0
5
10
15
20
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
λ^1
t=1
Steady State (t=62,574)

Conclusion

Well-motivated Rule of Thumb

- Optimizing Agent
  - Tension between direct and social observation
  - Imperfect communication $\implies$ Inference problem
- Solution to replicate direct observation
  - Inductive assumptions are “scientific”

Desirable Properties

- Tends to reach consensus (w/ DeGroot as a special case)
- Can generate non-degen dist of beliefs in steady state
  - Even when all have same model, directly-observe same data