Kurt Weichselberger’s Contribution to IP

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ISIPTA ’17
- Kurt Weichselberger passed away at 7th February last year
- He participated in the first six ISIPTAs, contributing papers, a tutorial and a special session
- **Look back at his work on IP**

[Photo kindly provided by Weichselberger’s family]
Introduction and Overview: The Project, Rudolf Seising

- Our work is embedded in a project studying the history of Statistics at LMU Munich.
- Rudolf Seising:
  - History of Science
  - LMU and German Museum Munich, currently temporary professor in Jena
  - also expert in history of fuzzy sets, soft computing
Kurt Weichselberger: Biographical Sketch

- *April 13, 1929, in Vienna
- 1953 PhD (Dr. Phil), supervised by Johann Radon
- Dep. of Statistics in Vienna (W. Winkler’s chair); social research institute in Dortmund; Cologne (J. Pfanzagls’ chair)
- 1962 Habilitation with a thesis on controlling census results
- 1967-68 university president (Rektor) TU Berlin
- from 1969 LMU Munich
- 1974 Foundation of the Institute of Statistics and Philosophy of Science at LMU
- from 1997 emeritus professor
- † February 7, 2016, in Grafing
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REKTORATSÜBERGABE

TECHNISCHE UNIVERSITÄT ULM

AN DER
TECHNISCHEN UNIVERSITÄT
ULM

Der Rektor Professor Dr. phil. Kurt Weichselberger
Background: Intensive discussion about Fisher’s fiducial argument

“[…] an attempt to eat the Bayesian omelette without breaking the Bayesian eggs” (Savage 1961, Proc 4th Berkeley)

“A comprehensive methodology of probabilistic modelling and statistical reasoning, which makes possible hierarchical modelling with information gained from empirical data. To achieve the goals of Bayesian approach – but without the pre-requisite of an assumed prior probability.”

(Source: Special session ISIPTA ’09, Durham (UK), p. 3)
Logical Probability as a Two-Place Function

Two-place function: $P(\text{conclusion} | \text{premise})$
Logical Probability as a Two-Place Function

Two-place function: \( P(\text{conclusion} \mid \text{premise}) \)

![Diagram showing the relationship between premise and conclusion with probability](image-url)
Logical Probability as a Two-Place Function

Two-place function: \( P(\text{conclusion} || \text{premise}) \)

- \text{premise}
- \text{conclusion}
- \text{model}
- \text{data}

Wahrscheinlichkeit
prove-ability

Rückschluss
(Inversion)
...We are challenged with the task to reconceptualise the foundations of probability. The question is whether we can make progress towards a broader concept designation without losing key benefits of the previous – objectivistic – concept.

[...] As in many cases in the history of science it is shown also here that – as a form of compensation for desired benefits – we have to abandon a “habit of thinking” (Denkgewohnheit). In the present case this is the habit of thinking that the probability is always a number. We must instead allow sets of numbers – say the interval between 0.2 and 0.3 – to act as the probability of the inference from the proposition B to the proposition A. [...] This extension of the probability concept from a number to a set of numbers is encouraged as soon as we try to formalize Fisher’s fiducial probability. Therefore, the American Henry Kyburg Jr. has already taken a similar approach [...] (Weichselberger, 1968, p. 47) [translation from German by TA & RS]
Overview of the talk

1. Social statistics; QC; TSAnal.
2. Logical prob. I
   [Wb, 1968]
3. Probability intervals
   [Wb & Pöhlmann, 1990]
4. Elementare Grundbegriffe
   [Wb, 2001]
5. Logical prob. II
   [Wb, 2005ff]
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Background

- Modelling uncertain knowledge
- Expert systems!

- Intensive discussion in computer science (AI)
- Big challenge for statistical methodology: traditional probabilities demand an unrealistic high level of precision and internal consistency
- Almost entirely ignored in statistics

- “Flexible uncertainty calculi” (e.g. MYCIN: certainty factors)
- Dempster Shafer Theory, combination rule
- Fuzzy Sets
Develop a probabilistically sound, flexible uncertainty calculus

[...An] argument against a possible application of probability theory [, understood in its traditional, precise form here,] in diagnostic systems is as follows: While probability theory affords statements, using real numbers as measures of uncertainty, the informative background of diagnostic systems is often not strong enough to justify statements of this type. [. . . ] However, it is possible to expand the framework of probability theory in order to meet these requirements without violating its fundamental assumptions. [. . . W]e believe that the weakness of estimates for measures of uncertainty as used in diagnostic systems represents a stimulus to enrich probability theory and the methodological apparatus derived from it, rather than an excuse for avoiding its theoretical claims. (Weichselberger and Pöhlmann, 1990, Springer LNAI, p. 2; emphasis by TA& RS)
Probability Intervals (PRIs):
Weichselberger & Pöhlmann (1990, Springer LNAI)

- **One-place probability**: probability of events
- Sample space \( \Omega = \{ \omega_1, \omega_2 \ldots \omega_k \} \)
- specify interval-valued assignments
  \[
  [L(E_i), U(E_i)]
  \]
  on the singletons \( E_i := \{ \omega_i \}, i = 1 \ldots, k \), only
- **R-PRI**: reasonable (\( \approx \) avoiding sure loss)
- **F-PRI**: feasible (\( \approx \) coherent)
- **derived PRI**: (\( \approx \) natural extension)

- “interval estimates” \( \rightarrow \) sensitivity analysis point of view
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1 Background and Historical Overview

2 Axioms

- Measurable space \((\Omega, \mathcal{A})\), assignments on \(\sigma\)-fields:
  \[
P(\cdot) = [L(\cdot), U(\cdot)]
  \]

- Structure \(\mathcal{M}\): set of all Kolmogorovian probabilities compatible with \(P(\cdot)\)

- \(R\)-probability: \(\mathcal{M} \neq \emptyset\) (\(\approx\) avoiding sure loss)

- \(F\)-probability: \(L(\cdot)\) and \(U(\cdot)\) are envelopes of \(\mathcal{M}\) (\(\approx\) coherence)
  \[
  L(A) = \inf_{p(\cdot) \in \mathcal{M}} p(A) \quad \text{and} \quad U(A) = \sup_{p(\cdot) \in \mathcal{M}} p(A), \quad \forall A \in \mathcal{A}.
  \]

- From \(R\)-probability to \(F\)-probability
  - rigorous standpoint (\(\approx\) natural extension)
  - cautious standpoint (\(\approx\) ?? )
3 Partially determinate probability
- Assessments on $\mathcal{A}_L, \mathcal{A}_U \subseteq \mathcal{A}$
- *Normal completion* ($\rightarrow$ natural extension)
- *Probability intervals*
- *Cumulative F-probability*: $\rightarrow$ p-boxes

4 Finite Spaces
- *Linear programming*
  - Checking R- and F-probability
  - Calculation of natural extension and of the assignment resulting from the cautious standpoint.
  - Duality theory is also powerful for deriving theoretical results
- *Generalized uniform probability/principle of insufficient reason*
  - *Epistemic Symmetry*: No knowledge of asymmetry (negative symmetry)
  - *Physical Symmetry*: Knowledge of symmetry (positive symmetry)
Activities 1991 to 2003 (Preparing the Book and after it)

- Melchsee-Frutt workshop
  - Walley, Goldstein, Hampel, Coolen, Morgenthaler, Smets...
- The first ISIPTAs
- Lev Utkin in Munich as Humboldt Fellow
- Colloquia on the occasions of 75th and 80th birthday

²photos kindly provided by Frank Coolen
Further Results on One-Place Probability

- manuscript of some 350 pages
- law of large numbers
- conditional probabilities: intuitive versus canonical concept
- Bayes’ theorem
- parametric models: interval-valued parameters
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Symmetrical Theory: Weichselberger (2009, V49, 268 pages)

I The Logical Concept of Probability
- W-fields: Axioms SI, SII
- Independence

II Duality
- concordant W-fields
- Axiom SIII
  - perfect duality
- applications in the classical context

III Inference
- regression
- preliminary: concatenation of W-fields
- preliminary: quasi-concordance
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Concluding Remarks

- Still challenging research program
- Embed it into/combine with current developments
- Attempts to build up a memorial page !?
- Archive office and private estate
Overview of the talk

1. 1960: Social statistics; QC; TSAnal.
   - Logical prob. I [Wb, 1968]
2. 1970: ?
5. 2010: Logical prob. II [Wb, 2005ff]

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