Comparison of Inference Relations Defined Over Different Sets of Ranking Functions

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Motivation

- $\mathcal{R}$ set of conditionals

- $\mid\!\sim_{\mathcal{R}}$ induced inference relation: $A \mid\!\sim_{\mathcal{R}} B$
- $\mid\!\sim_{\mathcal{R}}^{P}$ system P inference, p-entailment ... w.r.t. *all* models
- $\mid\!\sim_{\mathcal{R}}^{Z}$ system Z inference ... w.r.t. *single* model κ^z

- $\mid\!\sim_{\mathcal{R}}^{c}$ c-inference ... w.r.t. *all* c-representations
- $\mid\!\sim_{\mathcal{R}}^{c,u}$ c-inference ... w.r.t. *subset of* c-representations

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- $\mid\!\sim^{Z}_{\mathcal{R}}$ system Z inference ... w.r.t. *single* model κ^z
- $\mid\!\sim^{c}_{\mathcal{R}}$ c-inference ... w.r.t. *all* c-representations
- $\mid\!\sim^{c,u}_{\mathcal{R}}$ c-inference ... w.r.t. *subset of* c-representations
- $\mid\!\sim^{O}_{\mathcal{R}}$ inference w.r.t. *set of models*
- $\mid\!\sim^{O}_{\mathcal{R}} \overset{?}{=} \mid\!\sim^{O'}_{\mathcal{R}}$
- $\mid\!\sim^{c}_{\mathcal{R}} \overset{?}{=} \mid\!\sim^{c,u}_{\mathcal{R}}$

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- Conditional Logic "*if A then usually B*"
  - Ranking Functions (OCFs)
  - c-representations *calculated via CSP*
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Open Question: How to compare two skeptical inference relations defined over two sets of ranking functions?

Conditionals and OCFs [Spohn]

- $(B|A)$ with $A, B \in \mathcal{L}_\Sigma$
- $\mathcal{R} = \{r_1, \ldots, r_n\}$

$$\chi_{(B|A)}(\omega) = \begin{cases} ver. & \text{if} \quad \omega \models AB \\ fal. & \text{if} \quad \omega \models A\overline{B} \\ n.a. & \text{if} \quad \omega \models \overline{A} \end{cases}$$

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$$\kappa \models (B|A) \quad \text{iff} \quad \kappa(AB) < \kappa(A\overline{B}) \quad \text{iff} \quad A \mathrel{|\!\sim}_{\kappa} B$$

$$\kappa \models \mathcal{R} \quad \text{iff} \quad \kappa \models r_i \quad \text{for} \quad 1 \leqslant i \leqslant n$$

C-Representations [Kern-Isberner]

- For $\mathcal{R} = \{r_1, \ldots, r_n\}$
- $\eta_i \in \mathbb{N}$
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$\kappa_{\vec{\eta}} \models \mathcal{R}$ if η_i are solution to $CR(\mathcal{R})$ given as

$$\eta_i \geqslant 0$$

$$\eta_i > \min_{\omega \models A_i B_i} \sum_{\substack{j \neq i \\ \omega \models A_j \overline{B_j}}} \eta_j - \min_{\omega \models A_i \overline{B}_i} \sum_{\substack{j \neq i \\ \omega \models A_j \overline{B_j}}} \eta_j$$

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$\kappa_{\vec{\eta}} \models \mathcal{R}$ if η_i are solution to $CR^u(\mathcal{R})$ given as [IEA/AEI-17]

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$$\eta_i \leqslant u$$

sufficient $CR^u(\mathcal{R})$

skeptical inference over c-representations [FoIKS-16,ECAI-16]

$\mid\!\sim^{c}_{\mathcal{R}}$

taking every c-representation into account

$\mid\!\sim^{c,u}_{\mathcal{R}}$

taking every c-representation with **maximal impact** u into account

$CR^u(\mathcal{R})$ is *sufficient* **iff** $\mid\!\sim^{c}_{\mathcal{R}} = \mid\!\sim^{c,u}_{\mathcal{R}}$

sufficient $CR^u(\mathcal{R})$

$$\mathcal{R} = \{(a_1|\top), (a_2|\top)\} \qquad \text{over} \quad \Sigma = \{a_1, a_2\}$$

$$\vec{\eta}^{(1)} = (1,1) \quad \vec{\eta}^{(2)} = (1,2) \quad \vec{\eta}^{(3)} = (2,1) \quad \vec{\eta}^{(4)} = (2,2)$$

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$\Rightarrow CR^1(\mathcal{R})$ **is sufficient**

Idea

Model skeptical inference over a set of ranking models

by a single order relation on possible worlds

Comparing skeptical Inference Relations

Base Conditional

$$(\omega_1|\omega_1 \vee \omega_2)$$

Note: $\kappa \models (\omega_1|\omega_1 \vee \omega_2)$ iff $\kappa(\omega_1) < \kappa(\omega_2)$

Merged Order for set of ranking models O

$$<_O = \{(\omega_1, \omega_2) \mid \omega_1 \neq \omega_2, \kappa(\omega_1) < \kappa(\omega_2) \text{ for all } \kappa \in O\}$$

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Inference Core for inference relation $\mid\!\sim$

$$[\![\mid\!\sim]\!] = \{(\omega_1, \omega_2) \mid \omega_1 \vee \omega_2 \mid\!\sim \omega_1\}$$

ω	$\kappa_{\vec{\eta}(1)}(\omega)$	$\kappa_{\vec{\eta}(2)}(\omega)$	$\kappa_{\vec{\eta}(3)}(\omega)$	$\kappa_{\vec{\eta}(4)}(\omega)$
$\omega_0 : a_1\, a_2$	0	0	0	0
$\omega_1 : a_1\, \overline{a_2}$	1	2	1	2
$\omega_2 : \overline{a_1}\, a_2$	1	1	2	2
$\omega_3 : \overline{a_1}\, \overline{a_2}$	2	3	3	4

$O = \{\kappa_{\vec{\eta}(1)}, \ldots, \kappa_{\vec{\eta}(4)}\}$

$\mid\!\sim^{O}_{\mathcal{R}}$: skeptical inference over O

Base Conditional: $c = (\omega_0 | \omega_0 \vee \omega_1)$

$$\kappa_{\vec{\eta}(i)} \models c \quad \Rightarrow \quad \omega_0 \vee \omega_1 \mid\!\sim^{O}_{\mathcal{R}} \omega_0 \quad \Rightarrow \quad (\omega_0, \omega_1) \in [\![\mid\!\sim^{O}_{\mathcal{R}}]\!]$$

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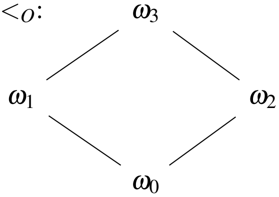
$<_O$:



Proposition: $[\![\mid\!\sim^{O}_{\mathcal{R}}]\!] = <_O$

Goal

Formulate criteria that allows us to compare inference relations

by comparing inference cores (i.e. only base conditionals)

$$\mid\!\sim^{O}_{\mathcal{R}} = \mid\!\sim^{O'}_{\mathcal{R}} \quad \text{iff} \quad [\![\mid\!\sim^{O}_{\mathcal{R}}]\!] = [\![\mid\!\sim^{O'}_{\mathcal{R}}]\!]$$

Merged Order Inference and Merged Order Compatibility

Def: Merged Order Inference

$$A \mid\!\sim_{\mathcal{R}}^{<_O} B \quad \text{iff} \quad \begin{array}{l} \text{for all } \omega' \in \Omega_{A\overline{B}} \\ \text{there is a } \omega \in \Omega_{AB} \\ \text{such that } \omega <_O \omega' \end{array}$$

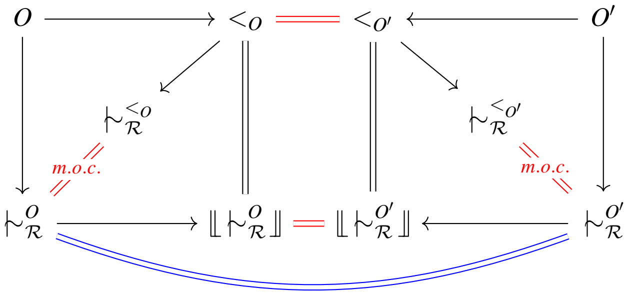
Proposition: for set of ranking functions O

$$\mid\!\sim_{\mathcal{R}}^{<_O} \subseteq \mid\!\sim_{\mathcal{R}}^{O} \quad \text{but} \quad \mid\!\sim_{\mathcal{R}}^{O} \not\subseteq \mid\!\sim_{\mathcal{R}}^{<_O}$$

Def: Merged Order Compatibility of O

$$\mid\!\sim_{\mathcal{R}}^{O} = \mid\!\sim_{\mathcal{R}}^{<_O}$$

Comparing skeptical Inference Relations



Legend:

(1.) definitions: $\rightarrow$

(2.) to show: $=$

(3.) consequence: $=$

Application

Conjecture: $CR^n(\mathcal{R})$ is sufficient for $n = |\mathcal{R}|$

Special Case: Sequence of KBs of conditional facts

$$\mathcal{R}_n = \{(a_1|\top), \ldots, (a_n|\top)\}$$
$$\text{over} \quad \Sigma_n = \{a_1, \ldots, a_n\}$$

Application

Sequence of KBs

- $\mathcal{R}_n = \{(a_1|\top), \ldots, (a_n|\top)\}$
- over $\Sigma_n = \{a_1, \ldots, a_n\}$

For $\mathcal{R}_n$

- $CR^{n-1}(\mathcal{R}_n)$ is sufficient

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By proving

- $O = \mathcal{O}(CR(\mathcal{R}_n))$ is m.o.c
- $O' = \mathcal{O}(CR^{n-1}(\mathcal{R}_n))$ is m.o.c
- $\llbracket \mathrel{|\!\sim}^{\mathcal{O}(CR^{n-1}(\mathcal{R}_n))}_{\mathcal{R}_n} \rrbracket = \llbracket \mathrel{|\!\sim}^{\mathcal{O}(CR(\mathcal{R}_n))}_{\mathcal{R}_n} \rrbracket$

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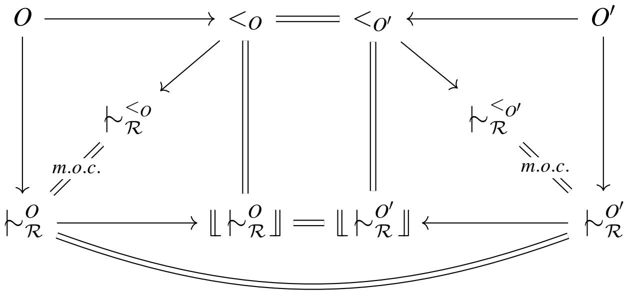

Conclusions and Future Work

- method for comparing skeptical inference relations
- applications to c-representations

- conditions for merged order compatibility
- determine sufficient upper bound for $CR^{u}(\mathcal{R})$ for arbitrary $\mathcal{R}$