A generic belief function model to handle multi-criteria preferences

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Where is Compiegne

Motivation

Many works on preferences under uncertainty:

- probability theory
- desirability
- prospect theory
- ...

Many works about uncertain (multi-criteria) preferences

- rank probabilistic models
- robust MCDM models
- random utility
- ...

Motivation: sequel

Recently, many works on collecting preference assessments to build robust (MCDM) preference models:

- version space, set-based approaches
- probabilistic approaches

Yet, few works on uncertainty in collected preferences (rather than in model). We do so by using belief functions:

- well-adapted to a non-statistical, fusion setting
- potential use of conflicting evidence to our advantage

A rather simple proposal

We assume

- A set of possible alternatives $\mathscr{X}$
- A version space $\mathscr{H}$ of possible preference models over $\mathscr{X}$:
  - Weighted averages, Choquet inegrals,
  - CP-nets,...
- Decision maker provides items $(\mathscr{I}_i, \alpha_i)$ where
  - $\mathscr{I}_i$: preference information (alternative comparisons, parameter assessments)
  - α_i: certainty degree about the provided information
- $\mathscr{I}_i$ can be mapped into a set $H_i \subseteq \mathscr{H}$ of compatible hypothesis

An example

- $\mathscr{X}$ =set of students
- Evaluated over
  - Physics (P) $\in [0,10]$
  - Math (M) $\in [0,10]$
  - French (F) $\in [0,10]$
- $\mathscr{H}$ = weighted averages
- Specified by (w_P, w_M, w_F)
  with $w_P + w_M + w_F = 1$

Assume two students $x_1 = (0,8,5)$ and $x_2 = (8,4,5)$, agent says $I_1 = \{x_1 \succ x_2\}$ with $\alpha_1 = 0.6$, then

$$0w_P + 8w_M + 5w_F > 8w_P + 4w_M + 5w_F \rightarrow w_M > 2w_P$$

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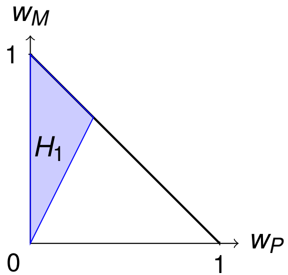
$$H_1 = \{(w_P, w_M) : w_M > 2w_P\}$$



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Mass functions and information combination

- Transform each item $(\mathscr{I}_i, \alpha_i)$ into a mass function m_i with

$$m_i(H_i) = \alpha_i \quad m_i(\mathscr{H}) = 1 - \alpha_i$$

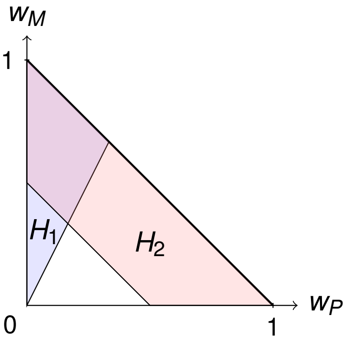
- Given two such masses m_1, m_2, combine them into

$$m_{1\cap 2}(H) = \sum_{H_i \in \mathscr{F}_i, H_1 \cap H_2 = H} m_1(H_1) m_2(H_2),$$

- The above equation being commutative and associative, extends to any number n of information
- Some mass can be given to $\emptyset$ in case of inconsistency

Example continued

- "Sciences more important than language"
- $w_P + w_M \geq w_F \rightarrow w_P + w_M \geq 0.5$
- $H_2 = \{(w_P, w_M) : w_P + w_M \geq 0.5\}$
- $\alpha_2 = 0.9$



The resulting mass is then

$$m(H_1) = 0.06,\ m(H_2) = 0.36,\ m(H_1 \cap H_2) = 0.54,\ m(\mathscr{H}) = 0.04.$$

Inferences: choice and ranking

- Each H_i defines a partial order P_i over set $\mathscr{X}$
- Given a subset $\mathscr{A} = \{a_1, \ldots, a_n\}$ of alternatives
  - Choice: recommend a best alternative a^*, or a subset A^*
  - Ranking: propose a partial ranking of alternatives

We will consider the following alternatives in our example:

	P	M	F
a_1	4	3	9
a_2	5	9	6

	P	M	F
a_3	8	7	3
a_4	7	1	7

$$P_1 = \{(a_1, a_4), (a_2, a_3)\},\ P_2 = P_{\mathscr{H}} = \{\},\ P_{1\cap 2} = \{(a_1, a_4), (a_2, a_1), (a_2, a_3)\}.$$

Choice

- Max_i denotes maximal elements of P_i
- Max_i= superset of A^*, maximal elements of the true underlying partial order
- Plausibility that a given subset A is a subset of A^*:

$$Pl(A \subseteq A^*) = \sum_{A \subseteq Max_i} m(H_i)$$

- $Pl(\{a\} \subseteq A^*) = 1$ only if $\{a\}$ maximal element of every P_i
- We can have $A \subseteq B$ with $Pl(A \subseteq A^*) \geq Pl(B \subseteq A^*)$
- Take subset with maximal plausibility

$$Max_1 = \{a_1, a_2\},\ Max_{1\cap 2} = \{a_2\},\ Max_2 = Max_{\mathscr{H}} = \mathscr{A}$$

	$\{a_1\}$	$\{a_2\}$	$\{a_3\}$	$\{a_4\}$	$\{a_1,a_2\}$	$\{a_1,a_3\}$	$\{a_1,a_4\}$	$\{a_2,a_3\}$	$\{a_2,a_4\}$	$\{a_3,a_4\}$
Pl	0.46	1	0.4	0.4	0.46	0.4	0.4	0.4	0.4	0.4

Ranking

- Compute for every pair the interval $[Bel(a_i \succ a_j), Pl(a_i \succ a_j)]$
- For a_i, get interval-valued score

$$[\underline{s}_i, \overline{s}_i] = \sum_{a_j \neq a_i} [Bel(a_i \succ a_j), Pl(a_i \succ a_j)]$$

- Rank according to the corresponding interval order

$$\begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{array} \begin{array}{c} \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \end{array} \\ \begin{pmatrix} 0 & [0, 0.46] & [0, 1] & [0.6, 1] \\ [0.54, 1] & 0 & [0.6, 1] & [0.54, 1] \\ [0, 1] & [0, 0.4] & 0 & [0.54, 1] \\ [0, 0.4] & [0, 0.46] & [0, 0.46] & 0 \end{pmatrix} \end{array} \overset{\sum}{=} \begin{array}{c} [\underline{s}_i, \overline{s}_i] \\ \begin{pmatrix} [0.6, 2.46] \\ [1.68, 3] \\ [0.54, 2.4] \\ [0, 1.32] \end{pmatrix} \end{array}$$

Proposed ranking: $P^* = \{(a_2, a_4)\}$

Conflicting information

Combination may lead to non-null mass $m(\emptyset)$ on empty set:

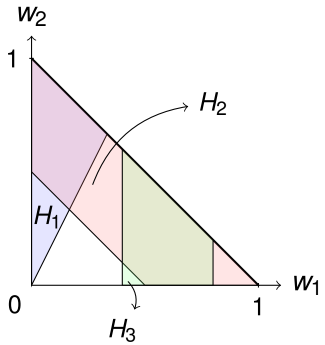
- due to inconsistent information given by DM
- **due to a too limited set of models $\mathscr{H}$**

Belief functions therefore interesting to solve these two issues by

- picking a subset of consistent information items
- **choosing an adequate space of models**

Choosing a model: example

- "Mathematics should account for $4/10$ to $8/10$ of the score"
- $0.8 \geq w_M \geq 0.4$
- $H_3 = \{(w_P, w_M) : 0.8 \geq w_M \geq 0.4\}$
- $\alpha_3 = 0.9$



The resulting mass on the empty set is

$$m(\emptyset) = 0.6 \cdot 0.9 \cdot 0.9 = 0.486$$

Model choice algorithm

Algorithm 1: Algorithm to select preference model

Input: Spaces $\mathscr{H}^1 \subseteq \ldots \subseteq \mathscr{H}^K$, Information $\mathscr{I}_1, \ldots, \mathscr{I}_F$, threshold τ, $i = 1$

Output: Selected hypothesis space $\mathscr{H}^*$

repeat

  foreach $j \in \{0, \ldots, m\}$ **do** Evaluate H_j^i;

  Combine $m_1^i, \ldots, m_F^i$ into m^i ;

  $i \leftarrow i + 1$

until $m^i(\emptyset) \leq \tau$ *or* $i = K + 1$;

Example continued

- $\mathscr{H}^i$= i-additive Choquet integral
- $\mathscr{H}^1$= weighted average, 3 parameters

$$\Rightarrow m(\emptyset) = 0.486$$

- $\mathscr{H}^2$= 2-additive, 6 parameters

$$\Rightarrow m(\emptyset) = 0$$

- $\mathscr{H}^2$ adequate model to represent provided preferences

Conclusions and perspectives

Our proposed model:

- easily integrates uncertainty in preference expression
- is quite generic regarding to the used model
- could be useful for information selection and/or model choice

The next steps are to

- instantiate it for some specific models (Choquet integrals, CP-net, . . .)
- define optimal elicitation strategies (in the line of Viappiani et al.)
- check that these latter do not suffer from same defect as similar strategies with certain answers
- connect them to Bayesian preference learning