A Transformation System for Unique Minimal Normal Forms of Conditional Knowledge Bases

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Overview

Logical Framework of Conditionals

Models for Conditionals and Knowledge Bases

A Transformation System for Conditional Knowledge Bases

Conclusion

Motivation: Why Normal Forms?

Normal forms

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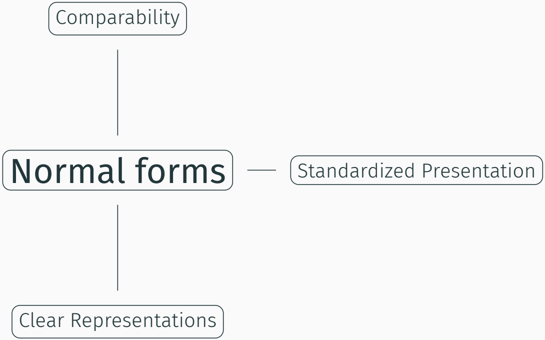
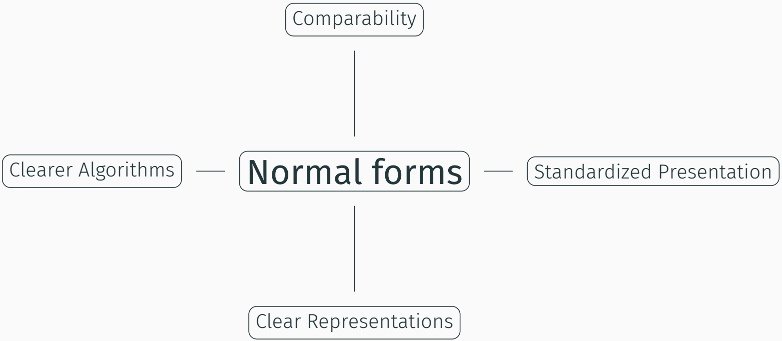
Comparability

Normal forms

Motivation: Why Normal Forms?


Comparability
Normal forms
Standardized Presentation
Clear Representations

Motivation: Why Normal Forms?



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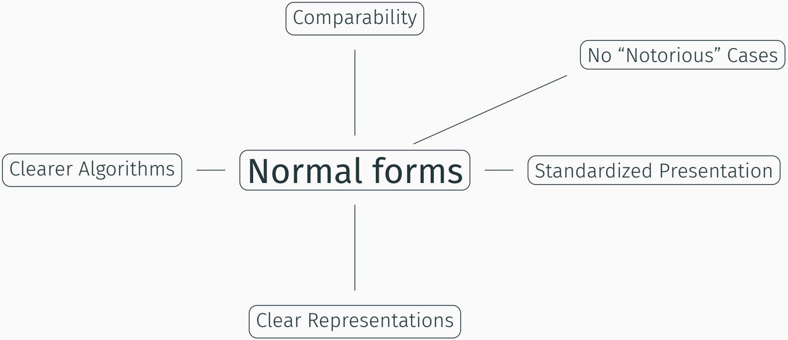

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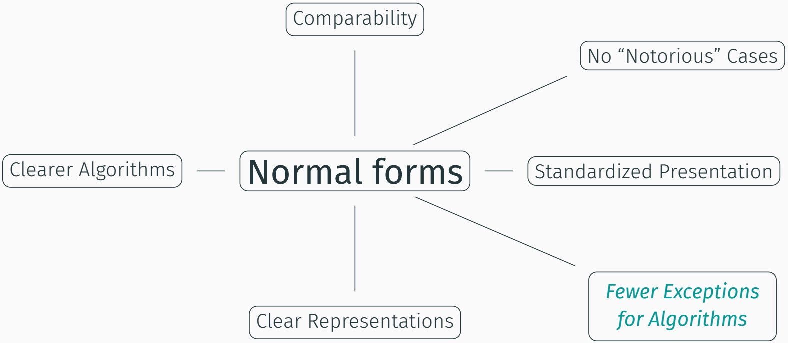

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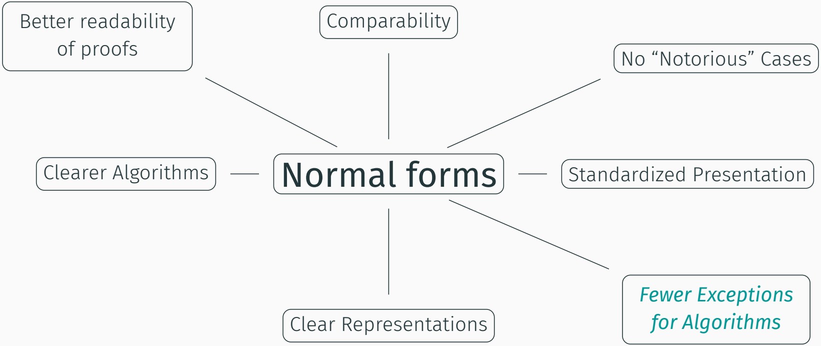

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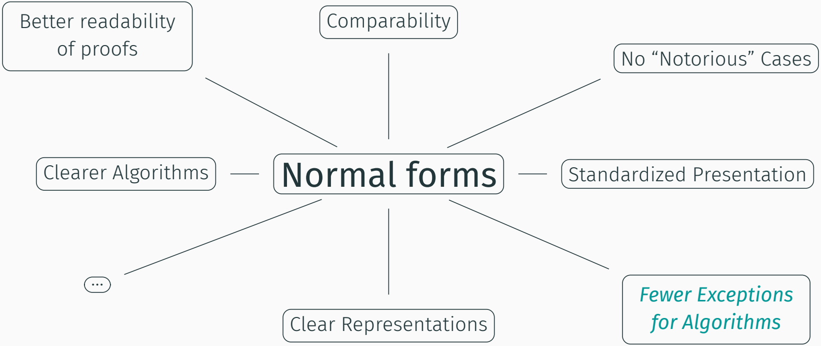

Logical Framework of Conditionals

We use a ***standard propositional logic*** with

- A finite propositional alphabet $\Sigma = \{V_1, \ldots, V_m\}$,
- Usual logical connectives $\wedge, \vee, \neg$, and
- A language $\mathfrak{L}$ of literals from Σ closed under these connectives.

We represent the set of ***possible worlds*** Ω syntactically with complete conjunctions of literals of Σ.

Example (Possible Worlds)

Let $\Sigma = \{C, E, F\}$ be the alphabet with variables for car, e-car and fossil fuel. The possible worlds for this alphabet are:

$$\Omega = \left\{ c\,e\,f,\ c\,e\,\bar{f},\ c\,\bar{e}\,f,\ c\,\bar{e}\,\bar{f},\ \bar{c}\,e\,f,\ \bar{c}\,e\,\bar{f},\ \bar{c}\,\bar{e}\,f,\ \bar{c}\,\bar{e}\,\bar{f} \right\}.$$

Conditionals and Conditional Knowledge Bases

- *Conditionals* $(B|A)$ encode defeasible rules *"If A then usually B"*.
- Three-valued evaluation by worlds [Fin74]:

$$[\![(B|A)]\!]_\omega = \begin{cases} \mathit{true} & \text{iff } \omega \models AB \quad (\text{"Rule verified"}) \\ \mathit{false} & \text{iff } \omega \models A\overline{B} \quad (\text{"Rule violated"}) \\ \mathit{undefined} & \text{iff } \omega \models \overline{A} \quad (\text{"Rule not applicable"}) \end{cases}$$

- Sets of conditionals $\mathcal{R} = \{(B_1|A_1), \dots, (B_n|A_n)\}$ are called *(conditional) knowledge bases*.

Example (Conditionals)

$(f\,|\,c)$: "Usually cars need fossil fuel."
$(\bar{f}\,|\,e)$: "Usually e-cars do not need fossil fuel."
$(c\,|\,e)$: "Usually e-cars are cars."

Models for Conditionals and Knowledge Bases

To give appropriate ***semantics to conditionals***, they are usually considered within *richer structures* such as ***epistemic states***.

Typical Models of Conditionals:

Quantitative	Qualitative	
Probabilistic	**Possibilistic**	**Plausibilistic**
P accepts $(B\|A)$ iff $P((B\|A)) = P(B\|A)$ "The Equation".	Π accepts $(B\|A)$ iff verification of $(B\|A)$ more *possible* than falsification of $(B\|A)$	κ accepts $(B\|A)$ iff verification of $(B\|A)$ more *plausible* than falsification of $(B\|A)$
$P \models (B\|A)$	$\Pi \models (B\|A)$	$\kappa \models (B\|A)$

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κ accepts $(B\|A)$ iff verification of $(B\|A)$ more *plausible* than falsification of $(B\|A)$

$$\kappa \models (B|A)$$

An Ordinal Conditional Function (OCF) or ***ranking function*** κ is a function that assigns a ***degree of disbelief*** to each world $\omega \in \Omega$.

Definition (OCF [Spohn '88])

$\kappa := \Omega \to \mathbb{N}_0^\infty$ such that:

$$\kappa^{-1}(0) \neq \varnothing$$

$$\kappa(A) = \min\{\kappa(\omega) | \omega \models A\}$$

$$\kappa(B|A) = \kappa(AB) - \kappa(A)$$

$$\kappa \models (B|A) \text{ iff } \kappa(AB) < \kappa(A\overline{B})$$

Example (Car Ranking)

$\overline{c}ef$	$\kappa(\omega) = 4$
$cef, \overline{c}e\overline{f}$	$\kappa(\omega) = 2$
$ce\overline{f}, c\overline{e}\overline{f}$	$\kappa(\omega) = 1$
$c\overline{e}f, \overline{c}\overline{e}f, \overline{c}\overline{e}\overline{f}$	$\kappa(\omega) = 0$

For knowledge bases we have $\kappa \models \mathcal{R}$ iff $\kappa \models (B|A)$ for all $(B|A) \in \mathcal{R}$.

Models & Equivalence (Intuition)

Classical and conditional models and equivalence

- A ***model*** of ***formula*** is a ***world***
  in which the ***formula*** is ***satisfied***.

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Classical and conditional models and equivalence

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Classical and conditional models and equivalence

- A *model* of *conditional* is an *epistemic state* in which the *conditional* is *accepted*.
- A *conditional* is *equivalent* to another *conditional* iff both have identical *models*.

Example

For instance, $\{(B|A)\} \quad \equiv \quad \{(BA|A)\}$

Two Notions of Equivalence for Conditional Knowledge Bases

Two conditional knowledge bases $\mathcal{R}$ and $\mathcal{R}'$ are

- *Elementwise equivalent* iff
  each conditional in $\mathcal{R}$
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- *Elementwise equivalent* iff
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- *Modelwise equivalent* iff
  they have the same models, i.e.,
  $\kappa \models \mathcal{R}$ iff $\kappa \models \mathcal{R}'$.

A Transformation System for Conditional Knowledge Bases

Intuitively, redundant information should be removed by a normalization, so, let, for instance,

$$\mathcal{R} = \left\{ \begin{array}{llll} r_1 = (f|c), & r_2 = (\bar{f}|e), & r_3 = (c|e), & r_4 = (e|e\bar{f}), \\ r_5 = (e\bar{f}|e), & r_6 = (\bar{e}|\top), & r_7 = (cf \vee \bar{c}f|ce \vee c\bar{e}) & \end{array} \right\}$$

Motivating Example

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$$\mathcal{R} \xrightarrow{\text{(normalize)}} \{r_1, r_3, r_4, r_5, r_6, r_7\}, \text{ since } e\bar{f} \equiv ee\bar{f}$$

Transformation System $\mathcal{T}$ (Intuition)

Define a transformation System $\mathcal{T}$ with rules $\mathfrak{r}$ such that

- $\mathfrak{r}(\mathcal{R})$ contains information ***equivalent*** to $\mathcal{R}$.
- $\mathcal{T}$ is ***terminating***
  (i.e. there is a fixed point Q s.t. $\mathcal{T}^{Q}(\mathcal{R}) = \mathcal{T}^{Q+1}(\mathcal{T})$).
- Order of rules application is irrelevant (***confluence***).
- $\mathcal{T}$ is ***minimizing***[1] (i.e. $|\mathcal{R}| \leq |\mathcal{T}(\mathcal{R})|$).

Then $\mathcal{T}^{Q}(\mathcal{R}) = \mathcal{T}(\mathcal{R})$ is a normal form of $\mathcal{R}$.

[1] With respect to given semantics

Transformation System $\mathcal{T}$ (Rules)

(SF)	self-fulfilling	$\dfrac{\mathcal{R} \cup \{(B\|A)\}}{\mathcal{R}}$	$A \models B, A \not\equiv \bot$
(DP)	duplicate	$\dfrac{\mathcal{R} \cup \{(B\|A), (B'\|A')\}}{\mathcal{R} \cup \{(B\|A)\}}$	$A \equiv A'$, $B \equiv B'$
(CE)	conditional equivalence	$\dfrac{\mathcal{R} \cup \{(B\|A), (B'\|A')\}}{\mathcal{R} \cup \{(B\|A)\}}$	$AB \equiv A'B'$, $A\overline{B} \equiv A'\overline{B'}$
(PN)	propositional normal form	$\dfrac{\mathcal{R} \cup \{(B\|A)\}}{\mathcal{R} \cup \{(\nu(B)\|\nu(A))\}}$	$A \neq \nu(A)$, $B \neq \nu(B)$
(CN)	conditional normal form	$\dfrac{\mathcal{R} \cup \{(B\|A)\}}{\mathcal{R} \cup \{(\nu(AB)\|\nu(A))\}}$	$B \neq AB$
(CC)	counter conditional	$\dfrac{\mathcal{R} \cup \{(B\|A), (\overline{B}\|A)\}}{\diamond}$	
(SC)	self contradictory	$\dfrac{\mathcal{R} \cup \{(B\|A)\}}{\diamond}$	$AB \equiv \bot$
(IC)	inconsistency	$\dfrac{\mathcal{R} \cup \{(B\|A)\}}{\diamond}$	$\mathcal{R} \neq \diamond$, $\Pi(\mathcal{R}) = \diamond$

Applying $\mathcal{T}$, the conditional knowledge base

$$\mathcal{R} = \left\{ \begin{array}{llll} r_1 = (f|c), & r_2 = (\bar{f}|e), & r_3 = (c|e), & r_4 = (e|e\bar{f}), \\ r_5 = (e\bar{f}|e), & r_6 = (\bar{e}|\top), & r_7 = (cf \vee \bar{c}f|ce \vee c\bar{e}) & \end{array} \right\}$$

is normalized to

$$\mathcal{R} = \left\{ \begin{array}{llll} r_1 = (f|c), & & r_3 = (c|e), & \\ r_5 = (e\bar{f}|e), & r_6 = (\bar{e}|\top), & & \end{array} \right\}$$

Properties of $\mathcal{T}$

Proposition

The transformation system $\mathcal{T}$ is

- terminating.
- correct.
- confluent.
- minimizing (wrt elementwise equivalence).

Proposition

The transformation system $\mathcal{T}$ is

- **terminating**.
- **correct**.
- **confluent**.
- minimizing (wrt elementwise equivalence).

Adding a rule that removes conditionals r_i that are System P entailable from $\mathcal{R} \setminus \{r_i\}$ gives us the property *minimizing* for model equivalence, with this we lose confluence.

Conclusion

Two types of equivalence for conditional knowledge bases

- *Via models*, like in propositional logic
  (Different model types possible, here: *OCF*)
  - *Elementwise* via models of conditionals or
  - *Modelwise* via models of conditional knowledge bases

Normal form for conditional knowledge base

- Set of model preserving rules $\rightarrow$ *transformation system* $\mathcal{T}$.
- (*Minimizing*)[2], *terminating*, *correct*, *confluent*.

$\Rightarrow$ $\mathcal{T}(\mathcal{R})$ is a (canonical)[2] normal form of $\mathcal{R}$.

[2] wrt elementwise equivalence

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$\Rightarrow$ $\mathcal{T}(\mathcal{R})$ is a (canonical)[2] normal form of $\mathcal{R}$.

Thank you for your attention.

[2] wrt elementwise equivalence

Literature

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