A Recourse Approach for the Capacitated Vehicle Routing Problem with Evidential Demands

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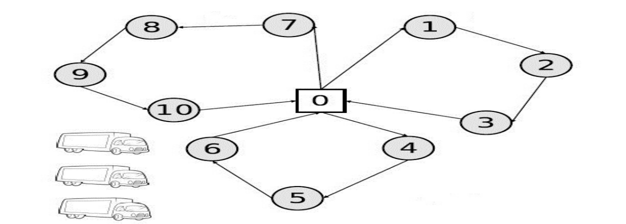
Background - CVRP / CVRPSD

The Capacitated Vehicle Routing Problem (CVRP)

- Finding the least cost routes to serve customers deterministic demands while respecting problem constraints, in particular vehicles capacity constraints.



The CVRP with Stochastic Demands (CVRPSD)

- Customers have stochastic demands.

The CVRPSD may be addressed by two main approaches:

- Chance Constrained Programming (CCP).
- Stochastic Programming with Recourse (SPR).

Alternative uncertainty framework

- In a previous work [1], the CVRP with Evidential Demands (CVRPED) modelled by a *belief function* based extension of CCP.
- *In this paper, we model the CVRPED by a belief function based extension of the SPR and solve it using a metaheuristic algorithm.*
- The first papers that handle discrete NP-hard problem involving uncertainty represented by belief functions.

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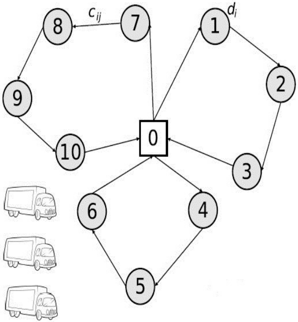
Outline

1. CVRP with Stochastic Demands
   - The CVRP
   - The CVRPSD modelled by SPR
2. CVRP with Evidential Demands
   - Belief function theory
   - The CVRPED modelled by a recourse approach
     - Formalisation
     - Uncertainty on recourses (failure situations)
     - Interval Demands
   - Experiments
3. Conclusions & perspectives

The CVRP

Given:

n = number of customers including the depot,
m = number of vehicles,
Q = vehicle capacity,
d_i = (known) demand of client i,
$c_{i,j}$ = cost of travelling from client i to client j,
$w_{i,j,k} = \left\{ \begin{matrix} 1 & \text{if } k \text{ travels from } i \text{ to } j, \\ 0 & \text{otherwise.} \end{matrix} \right\}$,
R_k = the route associated to vehicle k.



Objective function: $\min \sum_{k=1}^{m} C(R_k)$,

where: $C(R_k) = \sum_{i=0}^{n} \sum_{j=0}^{n} c_{i,j} w_{i,j,k}$, the travel cost of route R_k.

The CVRPSD

- d_i represents the stochastic demand of i (cannot exceed Q).
- Need to verify the capacity constraints of the CVRPSD for all realizations of $d_i \Rightarrow$ unrealistic.

A SPR approach for the CVRPSD

- Clients demands are collected until remaining vehicle capacity is not sufficient to pick up entire customer demand $\Rightarrow$ failure.
- If failure $\Rightarrow$ recourse (a return trip to the depot).
- Failure can happen at multiple customers except the first one.

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The CVRPSD modelled by SPR

The objective function becomes:

$$\min \sum_{k=1}^{m} C_{\mathrm{E}}(R_k),$$

where $C_{\mathrm{E}}(R_k)$ is the expected cost of R_k defined by

$$C_{\mathrm{E}}(R_k) = C(R_k) + C_{\mathrm{P}}(R_k),$$

with

- $C(R_k)$ the travel cost on R_k when no recourse action is performed;
- $C_{\mathrm{P}}(R_k)$ the expected penalty cost on R_k induced by failures.

Needed concepts

- A variable x taking values in a finite domain X.
- A MF $m^X : 2^X \to [0, 1]$ s.t. $\sum_{A \subseteq X} m^X(A) = 1$.
- A variable whose true value is known in the form of a MF is called an *evidential variable*.
- Given a MF m^X and a function h: $X \to \mathbb{R}^+$, then the upper expected value of h relative to m^X is :

$$E^*(h, m^X) = \sum_{A \subseteq X} m^X(A) \max_{x \in A} h(x).$$

The CVRPED

d_i represents the evidential demand of i (cannot exceed Q).

The recourse approach: failure situations

- Suppose a route R having N customers.
- $r_i = \begin{Bmatrix} 1 & \text{if failure occurs at the } i\text{-th client on } R \\ 0 & \text{otherwise} \end{Bmatrix}$ and $r_1 = 0$.
- Possible failure situations on R represented by vectors $(r_2, r_3, \ldots, r_N) \in \Omega$ s.t. $\Omega = \{0,1\}^{N-1}$.

Cost and uncertainty of each failure situation $\omega \in \Omega$

- Cost of each $\omega \in \Omega$ determined by $g : \Omega \to \mathbb{R}^+$.
- The penalty cost upon failure on i is $2c_{0,i} \Rightarrow g(\omega) = \sum_{i=2}^{N} r_i 2c_{0,i}$.
- A MF m^Ω representing uncertainty on failure situations on R.

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A pessimistic attitude: penalty cost and upper expected cost

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- The Objective of the CVRPED: $\min \sum_{k=1}^{m} C^*_{\mathrm{E}}(R_k)$,
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  - Similarities with robust optimisation.
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Example: no uncertainty on clients demands

A route R with $N = 3$ clients;

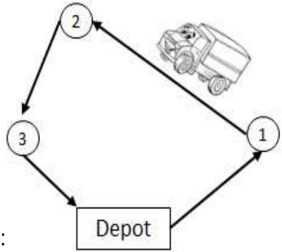
$\theta_1 = 3$ and $m_1(\theta_1) = 1$;

$\theta_2 = 3$ and $m_2(\theta_2) = 1$;

$\theta_3 = 5$ and $m_3(\theta_3) = 1$;

Capacity limit $Q = 5$;

$q_i, i = 1, \dots, N$, the vehicle load after serving i-th client:



- $r_1 = 0$ and $q_1 = \theta_1 = 3$.
- $r_2 = 1$ since $q_1 + \theta_2 > Q$, and $q_2 = q_1 + \theta_2 - Q = 1$.
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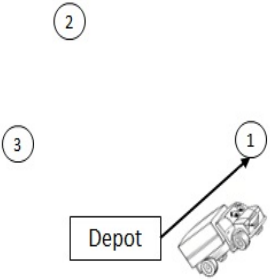

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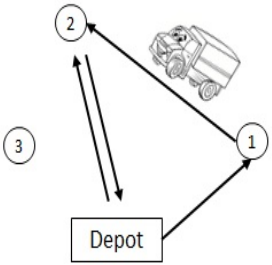

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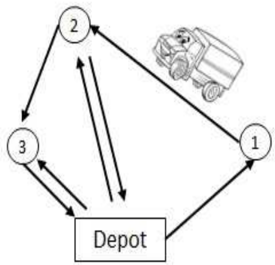

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Precise clients demands

- $f(\theta_1, \dots, \theta_N) = (r_2, r_3, \dots, r_N)$.

Imprecise clients demands

- MF m_i^Θ, $i = 1, \dots, N$, on R, s.t $m_i^\Theta(A_i) = 1$, $A_i \subseteq \Theta$.
- Then failure situation on R belongs to $B \subseteq \Omega$

$$B = f(A_1, \dots, A_N) = \bigcup_{(\theta_1,\dots,\theta_N)\in A_1\times\cdots\times A_N} f(\theta_1, \dots, \theta_N).$$

$m_i^\Theta, i = 1, \dots, N$ have arbitrary number of focal sets

The joint probability that $\theta_i \in A_i \subseteq \Theta, i = 1, \dots, N$ is

$$\prod_{i=1}^{N} m_i^\Theta(A_i) \Rightarrow \quad m^\Omega(B) = \sum_{f(A_1,\dots,A_N)=B} \prod_{i=1}^{N} m_i^\Theta(A_i).$$

Computing m^Ω: evaluating $f(A_1, \dots, A_N)$ for all combinations of focal sets of $m_i^\Theta \Rightarrow$ *worst-case complexity $\mathcal{O}(Q^N)$ (intractable).*

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Binary recourse tree example: route R with $N = 3$ clients

$\theta_1 \in [\![4;8]\!]$ and $m_1([\![4;8]\!]) = 1$;

$\theta_2 \in [\![5;7]\!]$ and $m_2([\![5;7]\!]) = 1$;

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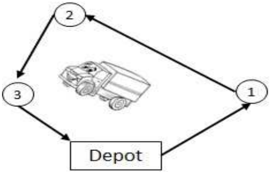

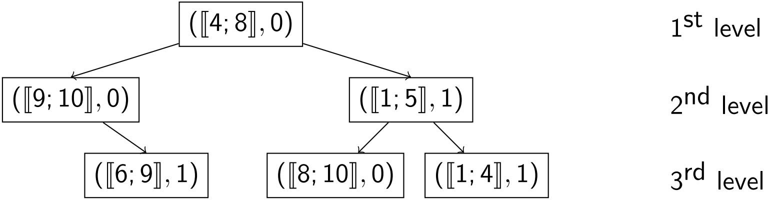

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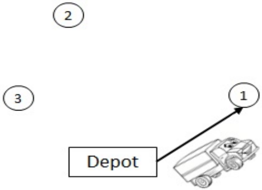

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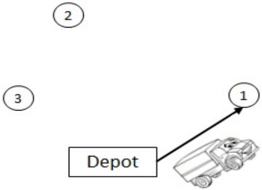

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$([\![4;8]\!], 0)$

1^{st} level

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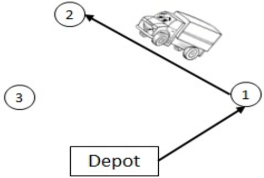

$$[\![4;8]\!] + [\![5;7]\!] = [\![9;15]\!] \Rightarrow \left\{ \begin{array}{ll} \text{no failure} & q_2 \in [\![9;10]\!] \\ \text{a failure} & q_2 \in [\![11-10;15-10]\!] \end{array} \right\}.$$

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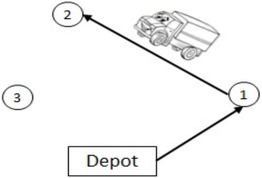

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$([\![4;8]\!], 0)$ — 1st level

$([\![9;10]\!], 0)$ $\quad$ $([\![1;5]\!], 1)$ — 2nd level

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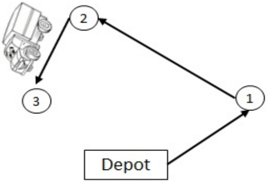
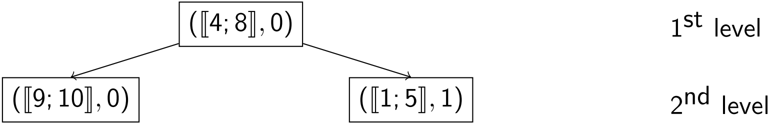


If $[\![9;10]\!] + [\![7;9]\!] = [\![16;19]\!] \Rightarrow$ a failure, $q_3 \in [\![16-10;19-10]\!]$,



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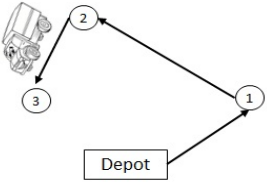
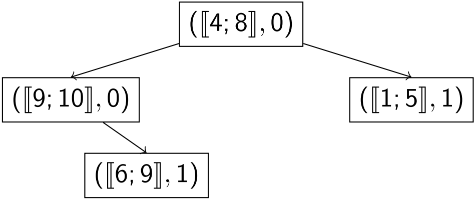


If $[\![9;10]\!] + [\![7;9]\!] = [\![16;19]\!] \Rightarrow$ a failure, $q_3 \in [\![16-10;19-10]\!]$,



1^{st} level

2^{nd} level

3^{rd} level

Binary recourse tree example: route R with $N = 3$ clients

$\theta_1 \in [\![4;8]\!]$ and $m_1([\![4;8]\!]) = 1$;

$\theta_2 \in [\![5;7]\!]$ and $m_2([\![5;7]\!]) = 1$;

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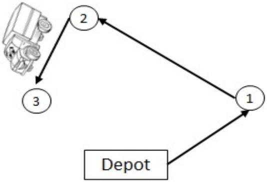
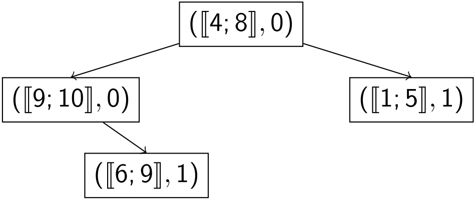


else $[\![1;5]\!] + [\![7;9]\!] = [\![8;14]\!] \Rightarrow \left\{\begin{matrix} \text{no failure} & q_3 \in [\![8;10]\!] \\ \text{a failure} & q_3 \in [\![11-10;14-10]\!] \end{matrix}\right\}$.



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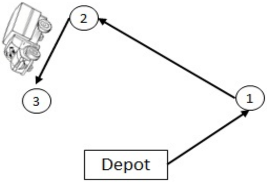

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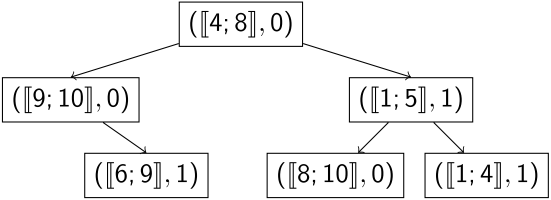

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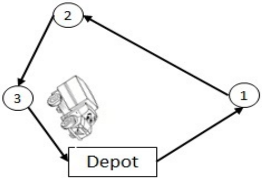
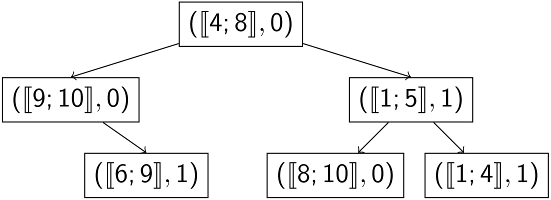


Worst case complexity is $\mathcal{O}(2^{N-1})$.



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Metaheuristic

Simulated annealing to solve the CVRPED via recourse.

CVRPED Benchmarks

Transformed each deterministic demand d^{det} in CVRP data sets, into an evidential demand with associated MF

$$\begin{aligned} m^{\Theta}(\{d^{det}\}) &= \alpha, \\ m^{\Theta}([\![\lfloor d^{det} - \gamma \cdot d^{det} \rfloor ; \lceil d^{det} + \gamma \cdot d^{det} \rceil]\!]) &= 1 - \alpha \end{aligned}$$

with $\alpha \in (0, 1)$ and $\gamma \in [0, 1]$.

Proposition

The optimal solution upper expected cost is non decreasing in γ

$\Rightarrow$ a lower bound on the optimal solution upper expected cost

$$\Leftrightarrow \quad \gamma = 0.$$

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Table: Simulated annealing results when $\alpha = 0.8$ and $\gamma = 0.1$

Instance	Best cost	Penalty cost	Avg cost	Stand. dev.	Avg runtime	Best cost $\gamma = 0$
A-n32-k5	843,06	0.03%	874,18	9,19	1837s.	839,18
A-n33-k5	705,69	0.37%	724,11	8,39	2241s.	697,12
A-n33-k6	773,55	0.75%	793,07	10,42	2271s.	758,36
A-n34-k5	820,37	1.40%	837,04	9,19	2975s.	812,16
A-n36-k5	884,51	0.34%	914,85	13,84	2715s.	869,10
A-n37-k5	722,57	0%	753,51	12,86	2634s.	720,85
A-n37-k6	1044,27	3.06%	1071,27	12,74	3111s.	995,07
A-n38-k5	781,69	8.36%	816,67	18,44	4525s.	748,64
A-n39-k5	890,88	1.57%	935,58	19	5068s.	885,04
A-n39-k6	896,60	0.34%	916,91	16.11	3196s.	884,09
A-n44-k6	1051,21	2.46%	1104,58	24,88	3922s.	1019,07
A-n45-k6	1091,72	6.01%	1129,21	18,98	5444s.	1006,90
A-n45-k7	1296,37	0.94%	1348,57	23,02	3237s.	1246,14
A-n46-k7	1060,47	0.05%	1087,16	16	2865s.	1045,93
A-n48-k7	1241,33	0.11%	1274,24	20,97	3119s.	1227,79

Conclusions

- The CVRPED modelled by an evidential extension of the SPR.
- A technique making computations tractable in realistic cases.
- Experiments using a simulated annealing algorithm.

Perspectives

- More recourse policies.
- Improving the solving algorithm.

References

N. Helal, F. Pichon, D. Porumbel, D. Mercier, and E. Lefèvre.
The capacitated vehicle routing problem with evidential demands: a belief-constrained programming approach.
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Thank you for your attention.