Reasoning in Description Logics with Typicalities and Probabilities of Exceptions

Gian Luca Pozzato

1Dipartimento di Informatica, Università degli Studi di Torino, Italy

ECSQARU 2017
DLs with Typicality and Probabilities

Outline

- Introduction to Description Logics
- DLs of Typicality
- Extensions with Probabilities
- Conclusions
DLs with Typicality and Probabilities

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Description Logics

Important formalisms of knowledge representation

Two key advantages:
- well-defined semantics based on first-order logic
- good trade-off between expressivity and complexity

at the base of languages for the semantic (e.g. OWL)

Knowledge bases

Two components:
- TBox = inclusion relations among concepts
- ABox = instances of concepts and roles = properties and relations among individuals
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- ABox = instances of concepts and roles = properties and relations among individuals
  - Platypus(perry)
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  - ABox = instances of concepts and roles = properties and relations among individuals
    - Platypus(perry)
Introduction

- TBox $\Rightarrow$ taxonomy of concepts
- need of representing prototypical properties and of reasoning about defeasible inheritance
- to handle defeasible inheritance needs the integration of some kind of nonmonotonic reasoning mechanism
  - DLs + MKNF
  - DLs + circumscription
  - DLs + default
- However, all these methods present some difficulties ...
Outline

DLs with typicality

- Non-monotonic extensions of Description Logics for reasoning about prototypical properties and inheritance with exceptions
  - Basic idea: to extend DLs with a typicality operator $T$
  - $T(C)$ singles out the “most normal” instances of the concept $C$
  - semantics of $T$ defined by a set of postulates that are a restatement of Lehmann-Magidor axioms of rational logic $R$

Basic notions

- A KB comprises assertions $T(C) \subseteq D$
- $T(Student) \subseteq FacebookUsers$ means “normally, students use Facebook”
- $T$ is nonmonotonic
  - $C \subseteq D$ does not imply $T(C) \subseteq T(D)$
### DLs with typicality

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The logic $\text{ALC} + \mathbf{T}_{\text{min}}$

**Example**

$\mathbf{T}(\text{Platypus}) \subseteq \neg \exists x \text{wears.Hat}$

$\mathbf{T}(\text{Platypus} \cap \text{SecretAgent}) \subseteq \exists x \text{wears.Hat}$

**Reasoning**

- ABox:
  - Platypus
  - Platypus \cap SecretAgent

- Expected conclusions:
  - wears.Hat (for perry)
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**Example**

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T(\text{Platypus}) \subseteq \neg \exists \text{wears.Hat}
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  - Platypus(perry)

- Expected conclusions:
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  - $Platypus(perry), SecretAgent(perry)$

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Reasoning

- ABox:
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The logic $\mathcal{ALC} + T$

Semantics

$\mathcal{M} = \langle \Delta^\mathcal{I}, <, .^\mathcal{I} \rangle$

- additional ingredient: preference relation among domain elements
- $<$ is an irreflexive, transitive, modular and well-founded relation over $\Delta^\mathcal{I}$:
  - for all $S \subseteq \Delta^\mathcal{I}$, for all $x \in S$, either $x \in Min_<(S)$ or $\exists y \in Min_<(S)$ such that $y < x$
  - $Min_<(S) = \{u : u \in S \text{ and } \not\exists z \in S \text{ s.t. } z < u\}$
- Semantics of the $T$ operator: $(T(C))^\mathcal{I} = Min_<(C^\mathcal{I})$
Weakness of monotonic semantics

Logic $\mathcal{ALC} + T$

- The operator $T$ is nonmonotonic, but...
- The logic is monotonic
  - If $KB \models F$, then $KB' \models F$ for all $KB' \supseteq KB$

Example

- In the KB of the previous slides:
  - if $\text{Platypus}(perry) \in \text{ABox}$, we are not able to:
  - assume that $T(\text{Platypus})(perry)$
  - infer that $\neg \exists w \text{wears}\ Hat(perry)$
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The nonmonotonic logic $\mathcal{ALC} + T_{\text{min}}$

Rational closure

- Preference relation among models of a KB
  - $\mathcal{M}_1 < \mathcal{M}_2$ if $\mathcal{M}_1$ contains less exceptional (not minimal) elements
  - $\mathcal{M}$ minimal model of KB if there is no $\mathcal{M}'$ model of KB such that $\mathcal{M}' < \mathcal{M}$
- Minimal entailment
  - $\mathcal{KB} \models_{\text{min}} F$ if $F$ holds in all minimal models of KB
- Nonmonotonic logic
  - $\mathcal{KB} \models_{\text{min}} F$ does not imply $\mathcal{KB}' \models_{\text{min}} F$ with $\mathcal{KB}' \supset \mathcal{KB}$
- Corresponds to a notion of rational closure of KB
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- Corresponds to a notion of rational closure of KB
**Introduction**

- \(\mathcal{ALC} + T^p\): extension of \(\mathcal{ALC}\) by typicality inclusions equipped by *probabilities of exceptionality*
- \(T(C) \subseteq_p D\), where \(p \in (0, 1)\)
- Intuitive meaning: typical Cs are also Ds with a probability \(p\) or normally, Cs are Ds and the probability of having exceptional Cs not being Ds is \(1 - p\)

**Example**

\(T(Student) \subseteq_{0.3} SportLover\)

\(T(Student) \subseteq_{0.9} SocialNetworkUser\)

- Sport lovers and social network users are both typical properties of students
- Probability of not having exceptions is 30% and 90%, respectively
**DLs + T and probabilities**

**Probabilistic DLs**

- $\mathcal{ALC} + T^P$ different from DLs with DISPONTE semantics
- Probabilistic axioms $p :: C \sqsubseteq D$ used to capture uncertainty
  - $Cs$ are $Ds$ with probability $p$
- In $\mathcal{ALC} + T^P$ typical properties to concepts and to reason about probabilities of exceptions to those typicalities
DLs + T and probabilities

Basic idea

- extensions of an ABox containing only some of the “plausible” typicality assertions of the rational closure of KB
  - each extension represents a scenario having a specific probability
  - probability distribution among scenarios
  - nonmonotonic entailment restricted to extensions whose probabilities belong to a given and fixed range
  - reason about scenarios that are not necessarily the most probable
DLs + $T$ and probabilities

Extensions of ABox

- typicality assumptions $T(C_1)(a_1), T(C_2)(a_2), \ldots, T(C_n)(a_n)$ inferred from $\mathcal{ALC} + T_{\text{min}}$
- extensions of ABox obtained by choosing *some* typicality assumptions
  - $\tilde{A}_1 = \{ T(C_1)(a_1), T(C_2)(a_2), \ldots, T(C_n)(a_n) \}$
  - $\tilde{A}_2 = \{ T(C_1)(a_1), T(C_2)(a_2), \ldots, T(C_n)(a_n) \}$
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  - $\ldots$
- reasoning in the *monotonic* $\mathcal{ALC} + T$ considering TBox and ABox extended with chosen assumptions
DLs + T and probabilities

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DLs + T and probabilities

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- reasoning in the \textit{monotonic $\mathcal{ALC} + \mathbf{T}$} considering TBox and ABox extended with chosen assumptions
Extensions of ABox and probabilities

\[ T(C) \sqsubseteq 0.3 \quad D \]
\[ T(C) \sqsubseteq 0.7 \quad E \]
\[ T(F) \sqsubseteq 0.8 \quad G \]
\[ T(C) \sqsubseteq 0.5 \quad H \]

\[ T(C)(a) \quad T(F)(a) \quad T(C)(b) \]
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\[ 0.3 \times 0.7 \times 0.5 \]
Extensions of ABox and probabilities

\[ T(C) \sqsubseteq_{0.3} D \quad T'(a) \quad T(F)(a) \quad T(C)(b) \]
\[ T(C) \sqsubseteq_{0.7} E \quad 0.105 \]
\[ T(F) \sqsubseteq_{0.8} G \quad 0.3 \times 0.7 \times 0.5 \]
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\[ 0.8 \]
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### Extensions of ABox and probabilities

<table>
<thead>
<tr>
<th>Description</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
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<tbody>
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\[ [0.105, 0.8, 0.105] \quad \widetilde{A}_1 \]

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- $[0.105, 0, 0]$: $\bar{A}_2$
- $[0, 0.8, 0.105]$: $\bar{A}_3$
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### Extensions of ABox and Probabilities

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<th>Probability</th>
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<tr>
<td>$T(F) \supseteq 0.8 , G$</td>
<td>$0.105 \quad 0.8 \quad 0.105$</td>
<td>$\mathbb{P}_{\mathcal{A}_3} = (1 - 0.105) \times 0.8 \times 0.105$</td>
</tr>
<tr>
<td>$T(C) \supseteq 0.5 , H$</td>
<td>$0.3 \times 0.7 \times 0.5$</td>
<td>$\mathbb{P}_{\mathcal{A}_8} = (1 - 0.105) \times (1 - 0.8) \times (1 - 0.105)$</td>
</tr>
</tbody>
</table>

- $\mathcal{A}_1$: $[0.105, 0.8, 0.105]$
- $\mathcal{A}_2$: $[0.105, 0, 0]$
- $\mathcal{A}_3$: $[0, 0.8, 0.105]$
- $\mathcal{A}_8$: $[0, 0, 0]$

Gian Luca Pozzato

Reasoning in DLs with Typicalities and Probabilities of Exceptions
**Entailment**

- Given $\text{KB}=(T, A)$ and $p, q \in (0, 1]$.
- $\mathcal{E} = \{\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_k\}$ set of extensions of $A$ whose probabilities are $p \leq \mathbb{P}_1 \leq q, p \leq \mathbb{P}_2 \leq q, \ldots, p \leq \mathbb{P}_k \leq q$.
- $\mathcal{T}' = \{T(C) \sqsubseteq D \mid T(C) \sqsubseteq r D \in \mathcal{T}\} \cup \{C \sqsubseteq D \in \mathcal{T}\}$.
- $\mathcal{KB} \models^{\langle p, q \rangle}_{\text{ALC}+\mathbb{P}} F$.
  - if $F$ is $C \sqsubseteq D$ or $T(C) \sqsubseteq D$, if $(\mathcal{T}', A) \models^{\text{ALC}+\text{min}}_{\mathbb{P}} F$.
  - if $F$ is $C(a)$, if $(\mathcal{T}', A \cup \tilde{A}_i) \models^{\text{ALC}+\mathbb{P}} F$ for all $\tilde{A}_i \in \mathcal{E}$.

- Probability of $F$: $\mathbb{P}(F) = \sum_{i=1}^{k} \mathbb{P}_i$. 

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Gian Luca Pozzato

Reasoning in DLs with Typicalities and Probabilities of Exceptions
DLs + T and probabilities

**Entailment**

- **Given** $\text{KB} = (\mathcal{T}, \mathcal{A})$ and $p, q \in (0, 1]$
- $\mathcal{E} = \{\widetilde{A}_1, \widetilde{A}_2, \ldots, \widetilde{A}_k\}$ set of extensions of $\mathcal{A}$ whose probabilities are
  $p \leq P_1 \leq q, p \leq P_2 \leq q, \ldots, p \leq P_k \leq q$
- $\mathcal{T}' = \{\mathbf{T}(C) \sqsubseteq D | \mathbf{T}(C) \sqsubseteq r D \in \mathcal{T}\} \cup \{C \sqsubseteq D \in \mathcal{T}\}$
- $\text{KB} \models^{(p,q)}_{\text{ALC+TP}} F$
  - if $F$ is $C \sqsubseteq D$ or $\mathbf{T}(C) \sqsubseteq D$, if $(\mathcal{T}', \mathcal{A}) \models^{\text{ALC+T}_{\text{min}}} F$
  - if $F$ is $C(a)$, if $(\mathcal{T}', \mathcal{A} \cup \widetilde{A}_i) \models^{\text{ALC+T}} F$ for all $\widetilde{A}_i \in \mathcal{E}$
- probability of $F$: $\mathbb{P}(F) = \sum_{i=1}^{k} P_i$
DLs + T and probabilities

Entailment

- Given $KB=(T, A)$ and $p, q \in (0, 1]$
- $E = \{\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_k\}$ set of extensions of $A$ whose probabilities are $p \leq P_1 \leq q, p \leq P_2 \leq q, \ldots, p \leq P_k \leq q$
- $T' = \{T(C) \subseteq D | T(C) \subseteq r D \in T\} \cup \{C \subseteq D \in T\}$
- $KB \models^{\langle p, q \rangle}_{\text{ALC+TP}} F$
  - if $F$ is $C \subseteq D$ or $T(C) \subseteq D$, if $(T', A) \models_{\text{ALC+T}_{\text{min}}} F$
  - if $F$ is $C(a)$, if $(T', A \cup \tilde{A}_i) \models_{\text{ALC+T}} F$ for all $\tilde{A}_i \in E$
- Probability of $F$: $P(F) = \sum_{i=1}^{k} P_i$
Entailment

- Given $KB = (T, A)$ and $p, q \in (0, 1]$
- $E = \{\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_k\}$ set of extensions of $A$ whose probabilities are $p \leq P_1 \leq q, p \leq P_2 \leq q, \ldots, p \leq P_k \leq q$
- $T' = \{T(C) \sqsubseteq D \mid T(C) \sqsubseteq r D \in T\} \cup \{C \sqsubseteq D \in T\}$
- $KB \models^{\langle p, q \rangle}_{ALC+T_P} F$
  - if $F$ is $C \sqsubseteq D$ or $T(C) \sqsubseteq D$, if $(T', A) \models_{ALC+T_{min}} F$
  - if $F$ is $C(a)$, if $(T', A \cup \tilde{A}_i) \models_{ALC+T} F$ for all $\tilde{A}_i \in E$
- Probability of $F$: $P(F) = \sum_{i=1}^{k} P_i$
Entailment

- Given \( KB = (T, A) \) and \( p, q \in (0, 1] \)
- \( E = \{ \widetilde{A}_1, \widetilde{A}_2, \ldots, \widetilde{A}_k \} \) set of extensions of \( A \) whose probabilities are \( p \leq P_1 \leq q, p \leq P_2 \leq q, \ldots, p \leq P_k \leq q \)
- \( T' = \{ T(C) \sqsubseteq D \mid T(C) \sqsubseteq_r D \in T \} \cup \{ C \sqsubseteq D \in T \} \)
- \( KB \models_{\mathcal{ALC}+T_P} F \)
  - if \( F \) is \( C \sqsubseteq D \) or \( T(C) \sqsubseteq D \), if \( (T', A) \models_{\mathcal{ALC}+T_{min}} F \)
  - if \( F \) is \( C(a) \), if \( (T', A \cup \widetilde{A}_i) \models_{\mathcal{ALC}+T} F \) for all \( \widetilde{A}_i \in E \)

- Probability of \( F \): \( P(F) = \sum_{i=1}^{k} P_i \)
**Description Logics**

**Description Logics of Typicality**

**DLs with Typicality and Probabilities**

**Conclusions**

**Typicalities and Probabilities**

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### DLs + T and probabilities

**TBox**

\[
\text{AtypicalDepressed} \sqsubseteq \text{Depressed}
\]

\[
\mathcal{T}(\text{Depressed}) \sqsubseteq 0.85 \neg \exists \text{Symptom}.\text{MoodReactivity}
\]

\[
\mathcal{T}(\text{AtypicalDepressed}) \sqsubseteq 0.6 \exists \text{Symptom}.\text{MoodReactivity}
\]

\[
\mathcal{T}(\text{ProstateCancerPatient}) \sqsubseteq 0.5 \exists \text{Symptom}.\text{MoodReactivity}
\]

\[
\mathcal{T}(\text{ProstateCancerPatient}) \sqsubseteq 0.8 \exists \text{Symptom}.\text{Nocturia}
\]

---

**Inferences**

- \( \mathcal{T}(\text{Depressed} \land \text{Tall}) \sqsubseteq \neg \exists \text{Symptom}.\text{MoodReactivity} \) is entailed in \( \mathcal{ALC} + \mathcal{T}^P \)

- if \( \mathcal{A} = \{ \text{ProstateCancerPatient}(\text{jim}), \text{AtypicalDepressed}(\text{jim}) \} \):
  - \( \exists \text{Symptom}.\text{MoodReactivity}(\text{jim}) \) has probability 76%
DLs + T and probabilities

**TBox**

\[ \text{AtypicalDepressed} \sqsubseteq \text{Depressed} \]

\[ \text{T(Depressed)} \sqsubseteq 0.85 \neg \exists \text{Symptom.MoodReactivity} \]

\[ \text{T(AtypicalDepressed)} \sqsubseteq 0.6 \exists \text{Symptom.MoodReactivity} \]

\[ \text{T(ProstateCancerPatient)} \sqsubseteq 0.5 \exists \text{Symptom.MoodReactivity} \]

\[ \text{T(ProstateCancerPatient)} \sqsubseteq 0.8 \exists \text{Symptom.Nocturia} \]

**Inferences**

1. \[ \text{T(Depressed} \sqcap \text{Tall)} \sqsubseteq \neg \exists \text{Symptom.MoodReactivity} \] is entailed in \( \mathcal{ALC} + T^P \)

2. if \( \mathcal{A} = \{ \text{ProstateCancerPatient(jim), AtypicalDepressed(jim)} \} \):
   - \( \exists \text{Symptom.MoodReactivity(jim)} \) has probability 76%
Reasoning Procedure

1: procedure ENTAILMENT((\mathcal{T}, \mathcal{A}), \mathcal{T}', F, \text{Tip}, p, q)
2: \text{Tip}_\mathcal{A} \leftarrow \emptyset \quad \triangleright \text{build the set } \mathcal{S} \text{ of possible assumptions}
3: \text{for each } C \in \text{Tip}_\mathcal{A} \text{ do}
4: \quad \text{for each individual } a \in \mathcal{A} \text{ do} \quad \triangleright \text{Reasoning in } ALC + T^{RaCl}_R
5: \quad \text{if } (\mathcal{T}', \mathcal{A}) \models_{ALC+T^{RaCl}_R} T(C)(a) \text{ then } \text{Tip}_\mathcal{A} \leftarrow \text{Tip}_\mathcal{A} \cup \{T(C)(a)\}
6: \quad \mathcal{P}_\mathcal{A} \leftarrow \emptyset \quad \triangleright \text{compute the probabilities of Definition 2 given } \mathcal{T} \text{ and } \text{Tip}_\mathcal{A}
7: \text{for each } C \in \text{Tip}_\mathcal{A} \text{ do}
8: \quad \Pi_C \leftarrow 1
9: \quad \text{for each } T(C) \sqsubseteq_p D \in \mathcal{T} \text{ do } \Pi_C \leftarrow \Pi_C \times p
10: \quad \mathcal{P}_\mathcal{A} \leftarrow \mathcal{P}_\mathcal{A} \cup \Pi_C
11: \mathcal{S} \leftarrow \text{build strings of possible assumptions as in Definition 3 given } \text{Tip}_\mathcal{A} \text{ and } \mathcal{P}_\mathcal{A}
12: \mathcal{E} \leftarrow \emptyset \quad \triangleright \text{build extensions of } \mathcal{A}
13: \text{for each } s_i \in \mathcal{S} \text{ do}
14: \quad \text{build the extension } \widehat{\mathcal{A}}_i \text{ corresponding to } s_i \text{ and compute } P_{\widehat{\mathcal{A}}_i} \text{ as in Definition 4}
15: \quad \text{if } p \leq P_{\widehat{\mathcal{A}}_i} \leq q \text{ then } \mathcal{E} \leftarrow \mathcal{E} \cup \widehat{\mathcal{A}}_i \quad \triangleright \text{select extensions with probability in } \langle p, q \rangle
16: \text{for each } \widehat{\mathcal{A}}_i \in \mathcal{E} \text{ do}
17: \quad \text{if } (\mathcal{T}', \mathcal{A} \cup \widehat{\mathcal{A}}_i) \not\models_{ALC+T^P_R} F \text{ then return } KB \not\models_{ALC+T^P_R} F \quad \triangleright \text{query entailment in } ALC + T^P_R
18: \text{return } KB \models_{ALC+T^P_R} F \quad \triangleright F \text{ is entailed in all extensions}
DLs + T and probabilities

Results

- entailment restricted to extensions with a fixed probability / range of probabilities
- essentially inexpensive
  - entailment in in \text{ExpTime} as in the underlying ALC
Beyond $\mathcal{ALC} + T^p$

**Future works**

- Combination of DLs with DISPONTE semantics with probability of exceptions
- Reasoning in real domains:
  - which range of probabilities?
- Implementation
- Extension to other DLs
References

Any question?

Perry The Platypus (aka Agent P)

Agent P (aka Perry The Platypus)