Exploiting Stability for Compact Representation of Independency Models

Linda C. van der Gaag

Department of Information and Computing Sciences,
Utrecht University, The Netherlands

Stavros Lopatatzidis

SYSTeMS Research Group,
Ghent University, Belgium

Recapturing independency models

An independency model over V is a set of triplets:

$$\langle A, B \mid C \rangle, \text{ with } A,B,C \subseteq V \text{ pairwise disjoint and } A,B \neq \varnothing$$

which is closed under the four semi-graphoid derivation rules:

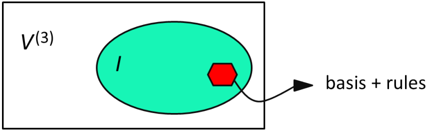
symmetry:	$\langle A,B \mid C \rangle \to \langle B,A \mid C \rangle$
decomposition:	$\langle A,B \mid C \rangle \to \langle A,B' \mid C \rangle$, for $B' \subseteq B$
weak union:	$\langle A, B_1 \cup B_2 \mid C \rangle \to \langle A, B_1 \mid C \cup B_2 \rangle$, for $B_1 \cap B_2 = \varnothing$
contraction:	$\langle A, B \mid C \cup D \rangle, \langle A, C \mid D \rangle \to \langle A, B \cup C \mid D \rangle$

This set of rules is sound, but not complete (Studený, 1992) !

Representing independency models

Independency models are represented by

- explicitly capturing just a basis of triplets;
- leaving all other triplets implicit through the derivation rules.



The state-of-the-art algorithm (Studený, 1998; Baioletti *et al.*, 2009)

- takes a starting set of triplets;
- uses a partial ordering on triplets, defined by the derivation rules;
- applies a tailored operator for constructing higher-ordered triplets;
- returns a basis of highest-ordered triplets.

Stable independencies

Stability = *"if two sets of variables are independent, they remain to be so regardless of any further information"*

A stable independency model over V is an independency model which is closed under the derivation rules:

- symmetry, decomposition, weak union and contraction
- strong union (De Waal, Van der Gaag, 2004):

  $$\langle A,B \,|\, C \rangle \rightarrow \langle A,B \,|\, C \cup D \rangle, \text{ for } D \subseteq V \setminus (A \cup B \cup C)$$

  strong contraction (Niepert, Van Gucht, Gyssens, 2010):

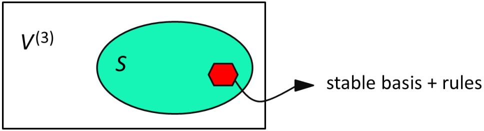
  $$\langle A,B \,|\, C \cup D \rangle, \langle A,B \,|\, C \cup E \rangle, \langle D,E \,|\, C \rangle \rightarrow \langle A,B \,|\, C \rangle$$

This set of rules is sound and complete for stable independency !

Representing stable independency models

Stable independency models can be represented by

- explicitly capturing a stable basis of triplets;
- leaving all other triplets implicit through the stable derivation rules.



For computing a stable basis, an algorithm should

- take a starting set of triplets;
- use a partial ordering on triplets, defined by the stable derivation rules;
- apply tailored operators for constructing higher-ordered triplets;
- return a basis of highest-ordered triplets.

Ingredients for stable basis computation

The main ingredients for our algorithm for computing a basis are:

- the notion of stable g-inclusion defines an ordering $\sqsubseteq_{\mathcal{S}}$ on triplets, covering symmetry, decomposition, weak union and strong union:

  $\langle A_1, B_1 \mid C_1 \rangle \sqsubseteq_{\mathcal{S}} \langle A_2, B_2 \mid C_2 \rangle$ if

  - $C_2 \subseteq C_1$;
  - $A_1 \subseteq A_2$ and $B_1 \subseteq B_2$, or $B_1 \subseteq A_2$ and $A_1 \subseteq B_2$.

- the operator $gc_{\mathcal{S}}(\cdot,\cdot)$ constructs higher-ordered triplets by means of the contraction rule (motivated by De Waal, Van der Gaag, 2004);

- the operator $gsc_{\mathcal{S}}(\cdot,\cdot,\cdot)$ constructs higher-ordered triplets by means of the strong contraction rule.

Some experimental results

Our preliminary experiments compare the two scenarios

1. using just the strong union derivation rule;
2. using both the strong union and strong contraction rules;

for stable basis computation, and reveal that

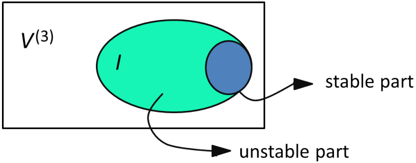
- for randomly generated starting sets, the scenarios return the same basis ($n = 10$);
- for starting sets allowing application of the $gsc_{\mathcal{S}}$-operator, the basis from scenario 1 is reduced by 40% on average ($n = 10$) by scenario 2.

Open question: Can scenario 2 result in a larger basis than scenario 1 ?

Revisiting independency models in general

An independency model is composed of

- a (possibly empty) stable part;
- a (possibly empty) unstable part.



In general,

- the stable part constitutes a stable independency model;
- the unstable part need not be an independency model.

Exploiting stability for representing models is not straightforward !

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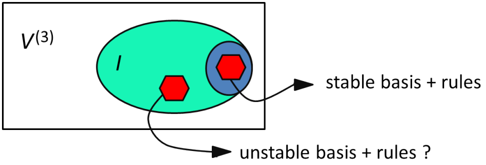

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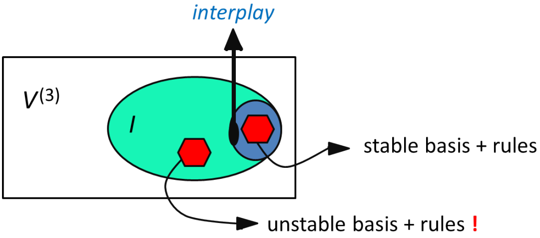
Representing independency models in general

Representing independency models by a stable basis and an unstable basis requires monitoring of their interplay (De Waal, Van der Gaag, 2004):



Our preliminary experiments indicate that introducing the $gsc_{\mathcal{S}}$-operator may result in a larger overall basis.

Further research

In our further research we will study

- the effects of the strong-contraction derivation rule;
- the interplay between the unstable and stable parts of an independency model;

and will identify, of an independency model, other parts with a highly regular structure.