1. Introduction & Take-home Message

Aumann (1976) gives sufficient conditions when two (precise) Bayesian agents with the same prior over a measurable space cannot agree to disagree on posteriors of a hypothesis H. Their posteriors for H must be equal if they are commonly known, along with their personal information sets leading to these posteriors.

- Geanakoplos and Polemarchakis (1982) extend Aumann’s result to a setting where the agents make their credences common knowledge by communication.
- We investigate Aumann’s result in a combination of these two more general settings, and show that the interesting and anomalous phenomenon known as dilation is the key obstacle to reaching agreements via communicating posteriors by Bayesian agents with imprecise priors.

2. Aumann’s Agreement Theorem

- Suppose two agents have the same (precise) prior, p, over a measurable space (Ω, A).
- Agent 1 learns (privately) the value of a partition of Ω, P1, and updates by Bayesian conditioning.
- All these are commonly known: each agent knows them, knows that each knows them, knows that each knows that each knows them, … ad infinitum.
- Then, if it is common knowledge that agent 1’s posterior of an event H is p1 and agent 2’s posterior of H is p2, then p1 = p2. That is, the agents cannot agree to disagree!

3. Example of Disagreement with IP

- Suppose p = (1/2, 1/4, 1/4), and suppose agent 1 learns [w1, w2] while agent 2 learns [w3, w4].
- So, p1([w1]) = p([w1, w2]) = 1/2. Similarly, p1([w3]) = 1/4, where p1 = p(· | w1).
- Note: it is common knowledge that p1([H]) = 1/2 and that p2([H]) = 1/3.

4. A Procedure of Communicating Imprecise Posteriors

Suppose the true state is w. The initial common knowledge is C0 = P(w), where P is the finest common coarsening of P1 and P2.

- Step 0: Agent i updates her credence of H to Q0(H) = Q(H | P1(w)). Let P0 = {E ∈ P | E ∩ C0 = ∅}.
- Step n + 1: They announce Q0n(H) and Q0n+1(H), respectively. Let
  \[ N_{n+1} = \{ E ∈ P_m | Q(H | E ∩ C_n) = Q(H) \} \]
  \[ C_{n+1} = \bigcup_{n=1}^{N_{n+1}} \bigcup_{n=1}^{N_{n+1}^2} \bigcup_{n=1}^{N_{n+1}^3} E \in P_{n+1} | E ∩ C_{n+1} = ∅ \} \]

- If P0 = Pm for (Cm+1 = Cm), neither agent learns new information and the procedure stops; otherwise, agent i updates credence of H to Q0n+1(H) = Q(H | Pn(w) ∩ Cn+1), and enters the next step.

5. A Generalization of Aumann’s Agreement Theorem

- In the absence of dilation, two agents are guaranteed to reach consensus on lower and upper probabilities by communicating posteriors. More formally:
  \[ \text{Theorem: Suppose the above procedure stops at step } n + 1 \text{. If for both } i = 1, 2, \{ E ∈ C_n | E ∈ P_m \} \text{ does not dilate } H, \text{ then } Q(H | P^i(w) ∩ C_n) = Q(H | P^j(w) ∩ C_n) \text{ and } Q(H | P(w) ∩ C_n) = Q(H | P(w) ∩ C_n). \]
- It is easy to show that the above result still holds if at each step the agents communicate only lower and upper probabilities, instead of the whole sets of probabilities.

6. Concluding Remarks and Further Questions

- The presence of dilation is necessary for agreeing to disagree on the lower or upper posterior.
- To put it differently, dilation-averse agents cannot agree to disagree on the lower or upper posterior.
- What about agents whose priors agree only partially?
- What about other updating rules, e.g., the Dempster-Shafer rule?