UNIVERSITÀ
DEGLI STUDI DI TRIESTE

Weak Dutch Books versus Strict Consistency with Lower Previsions

Chiara Corsato, Renato Pelessoni and Paolo Vicig

DEAMS - University of Trieste

ISIPTA '17, Lugano
July 12, 2017

Outline

Background

- (Williams') coherent and convex imprecise previsions

Weak Dutch Books

- Local precision properties with coherent and convex imprecise previsions
- Relationship between WDBs and positive probability events
- Vulnerability to real Dutch Books
- Hedging Weak Dutch Books

Additional reference:

✎ *C. Corsato, R. Pelessoni, P. Vicig, Weak Dutch Books with Imprecise Previsions, IJAR, 88:72–90, 2017.*

Notation

$\mathcal{D}$	set of conditional gambles $X\|B$	set of conditional events $E\|B$
$\underline{P} : \mathcal{D} \to \mathbb{R}$	lower prevision	lower probability
$\overline{P} : \mathcal{D} \to \mathbb{R}$	upper prevision	upper probability
Conjugacy relation	$\overline{P}(X\|B) = -\underline{P}(-X\|B)$	$\overline{P}(E\|B) = 1 - \underline{P}(\neg E\|B)$

Notation

$\mathcal{D}$	set of conditional gambles $X\|B$	set of conditional events $E\|B$
$\underline{P}: \mathcal{D} \to \mathbb{R}$	lower prevision	lower probability
$\overline{P}: \mathcal{D} \to \mathbb{R}$	upper prevision	upper probability
Conjugacy relation	$\overline{P}(X\|B) = -\underline{P}(-X\|B)$	$\overline{P}(E\|B) = 1 - \underline{P}(\neg E\|B)$

Notation

$\mathcal{D}$	set of conditional gambles $X\|B$	set of conditional events $E\|B$
$\underline{P} : \mathcal{D} \to \mathbb{R}$	lower prevision	lower probability
$\overline{P} : \mathcal{D} \to \mathbb{R}$	upper prevision	upper probability
Conjugacy relation	$\overline{P}(X\|B) = -\underline{P}(-X\|B)$	$\overline{P}(E\|B) = 1 - \underline{P}(\neg E\|B)$

Notation

$\mathcal{D}$	set of conditional gambles $X \vert B$	set of conditional events $E \vert B$
$\underline{P} : \mathcal{D} \to \mathbb{R}$	lower prevision	lower probability
$\overline{P} : \mathcal{D} \to \mathbb{R}$	upper prevision	upper probability
Conjugacy relation	$\overline{P}(X \vert B) = -\underline{P}(-X \vert B)$	$\overline{P}(E \vert B) = 1 - \underline{P}(\neg E \vert B)$

Notation

$\mathcal{D}$	set of conditional gambles $X\|B$	set of conditional events $E\|B$
$\underline{P} : \mathcal{D} \to \mathbb{R}$	lower prevision	lower probability
$\overline{P} : \mathcal{D} \to \mathbb{R}$	upper prevision	upper probability
Conjugacy relation	$\overline{P}(X\|B) = -\underline{P}(-X\|B)$	$\overline{P}(E\|B) = 1 - \underline{P}(\neg E\|B)$

Notation

$\mathcal{D}$	set of conditional gambles $X \mid B$	set of conditional events $E \mid B$
$\underline{P} : \mathcal{D} \to \mathbb{R}$	lower prevision	lower probability
$\overline{P} : \mathcal{D} \to \mathbb{R}$	upper prevision	upper probability
Conjugacy relation	$\overline{P}(X \mid B) = -\underline{P}(-X \mid B)$	$\overline{P}(E \mid B) = 1 - \underline{P}(\neg E \mid B)$

Notation

$\mathcal{D}$	set of conditional gambles $X\|B$	set of conditional events $E\|B$
$\underline{P} : \mathcal{D} \to \mathbb{R}$	lower prevision	lower probability
$\overline{P} : \mathcal{D} \to \mathbb{R}$	upper prevision	upper probability
Conjugacy relation	$\overline{P}(X\|B) = -\underline{P}(-X\|B)$	$\overline{P}(E\|B) = 1 - \underline{P}(\neg E\|B)$

Notions of *dF*-coherence, *W*-coherence and convexity

$\underline{P} : \mathcal{D} \to \mathbb{R}$ lower prevision.

Define, $\forall n \in \mathbb{N}^+$, $\forall X_i|B_i \in \mathcal{D}$, $\forall s_i \in \mathbb{R}$ $(i = 0, 1, \ldots, n)$,

$$\underline{G} = \sum_{i=1}^{n} s_i \, B_i\big(X_i - \underline{P}(X_i|B_i)\big) - s_0 \, B_0\big(X_0 - \underline{P}(X_0|B_0)\big), \quad B = \bigvee_{i=0}^{n} B_i,$$

and require

$$\sup(\underline{G}|B) \geq 0.$$

Notions of *dF*-coherence, *W*-coherence and convexity

$\underline{P} : \mathcal{D} \to \mathbb{R}$ lower prevision.

Define, $\forall n \in \mathbb{N}^+$, $\forall X_i|B_i \in \mathcal{D}$, $\forall s_i \in \mathbb{R}$ $(i = 0, 1, \dots, n)$,

$$\underline{G} = \sum_{i=1}^{n} s_i\, B_i\big(X_i - \underline{P}(X_i|B_i)\big) - s_0\, B_0\big(X_0 - \underline{P}(X_0|B_0)\big), \quad B = \bigvee_{i=0}^{n} B_i,$$

and require

$$\sup(\underline{G}|B) \geq 0.$$

- $\underline{P}$ *dF*-coherent: no constraints on $s_i \in \mathbb{R}$ $(i = 0, 1, \dots, n)$.
  (*de Finetti, 1974; Holzer, 1985*)

Notions of dF-coherence, W-coherence and convexity

$\underline{P} : \mathcal{D} \to \mathbb{R}$ lower prevision.

Define, $\forall n \in \mathbb{N}^+$, $\forall X_i|B_i \in \mathcal{D}$, $\forall s_i \in \mathbb{R}$ $(i = 0, 1, \ldots, n)$,

$$\underline{G} = \sum_{i=1}^{n} s_i \, B_i\big(X_i - \underline{P}(X_i|B_i)\big) - s_0 \, B_0\big(X_0 - \underline{P}(X_0|B_0)\big), \quad B = \bigvee_{i=0}^{n} B_i,$$

and require

$$\sup(\underline{G}|B) \geq 0.$$

- $\underline{P}$ dF-coherent: no constraints on $s_i \in \mathbb{R}$ $(i = 0, 1, \ldots, n)$.
  (*de Finetti, 1974; Holzer, 1985*)
- $\underline{P}$ W-coherent: $s_i \geq 0$ $(i = 0, 1, \ldots, n)$.
  (*Williams, 1975*)

Notions of dF-coherence, W-coherence and convexity

$\underline{P} : \mathcal{D} \to \mathbb{R}$ lower prevision.

Define, $\forall n \in \mathbb{N}^+$, $\forall X_i|B_i \in \mathcal{D}$, $\forall s_i \in \mathbb{R}$ $(i = 0, 1, \ldots, n)$,

$$\underline{G} = \sum_{i=1}^{n} s_i B_i\big(X_i - \underline{P}(X_i|B_i)\big) - s_0 B_0\big(X_0 - \underline{P}(X_0|B_0)\big), \quad B = \bigvee_{i=0}^{n} B_i,$$

and require

$$\sup(\underline{G}|B) \geq 0.$$

- $\underline{P}$ dF-coherent: no constraints on $s_i \in \mathbb{R}$ $(i = 0, 1, \ldots, n)$.
  (*de Finetti, 1974; Holzer, 1985*)
- $\underline{P}$ W-coherent: $s_i \geq 0$ $(i = 0, 1, \ldots, n)$.
  (*Williams, 1975*)
- $\underline{P}$ convex: $s_i \geq 0$ $(i = 1, \ldots, n)$ and $\sum_{i=1}^{n} s_i = s_0 > 0$.

Notions of dF-coherence, W-coherence and convexity

$\underline{P} : \mathcal{D} \to \mathbb{R}$ lower prevision.

Define, $\forall n \in \mathbb{N}^+$, $\forall X_i|B_i \in \mathcal{D}$, $\forall s_i \in \mathbb{R}$ $(i = 0, 1, \ldots, n)$,

$$\underline{G} = \sum_{i=1}^{n} s_i B_i\big(X_i - \underline{P}(X_i|B_i)\big) - s_0 B_0\big(X_0 - \underline{P}(X_0|B_0)\big), \quad B = \bigvee_{i=0}^{n} B_i,$$

and require

$$\sup(\underline{G}|B) \geq 0.$$

- $\underline{P}$ dF-coherent: no constraints on $s_i \in \mathbb{R}$ $(i = 0, 1, \ldots, n)$.
  (*de Finetti, 1974; Holzer, 1985*)
- $\underline{P}$ W-coherent: $s_i \geq 0$ $(i = 0, 1, \ldots, n)$.
  (*Williams, 1975*)
- $\underline{P}$ convex: $s_i \geq 0$ $(i = 1, \ldots, n)$ and $\sum_{i=1}^{n} s_i = s_0 > 0$.
  $\underline{P}$ centered convex: further, $\forall X|B \in \mathcal{D}$, $\varnothing|B \in \mathcal{D}$ and $\underline{P}(\varnothing|B) = 0$.
  (*Pelessoni, Vicig, 2005*)

Notions of dF-coherence, W-coherence and convexity

$\underline{P} : \mathcal{D} \to \mathbb{R}$ lower prevision.

Define, $\forall n \in \mathbb{N}^+$, $\forall X_i|B_i \in \mathcal{D}$, $\forall s_i \in \mathbb{R}$ $(i = 0, 1, \ldots, n)$,

$$\underline{G} = \sum_{i=1}^{n} s_i B_i \big(X_i - \underline{P}(X_i|B_i)\big) - s_0 B_0 \big(X_0 - \underline{P}(X_0|B_0)\big), \quad B = \bigvee_{i=0}^{n} B_i,$$

and require

$$\sup(\underline{G}|B) \geq 0.$$

- $\underline{P}$ dF-coherent: no constraints on $s_i \in \mathbb{R}$ $(i = 0, 1, \ldots, n)$.
  (*de Finetti, 1974; Holzer, 1985*)
- $\underline{P}$ W-coherent: $s_i \geq 0$ $(i = 0, 1, \ldots, n)$.
  (*Williams, 1975*)
- $\underline{P}$ convex: $s_i \geq 0$ $(i = 1, \ldots, n)$ and $\sum_{i=1}^{n} s_i = s_0 > 0$.
  $\underline{P}$ centered convex: further, $\forall X|B \in \mathcal{D}$, $\varnothing|B \in \mathcal{D}$ and $\underline{P}(\varnothing|B) = 0$.
  (*Pelessoni, Vicig, 2005*)

In each case, $\underline{G}|B$ is an admissible gain for $\underline{P}$.

What is a Weak Dutch Book gain?

Let $\underline{P} : \mathcal{D} \to \mathbb{R}$ be a convex (W-coherent) lower prevision. Any admissible gain $\underline{G}|B$ for $\underline{P}$ satisfies

$$\sup(\underline{G}|B) \geq 0.$$

What is a Weak Dutch Book gain?

Let $\underline{P} : \mathcal{D} \to \mathbb{R}$ be a convex (W-coherent) lower prevision. Any admissible gain $\underline{G}|B$ for $\underline{P}$ satisfies

$$\sup(\underline{G}|B) \geq 0.$$

Weak Dutch Book gain

$\underline{G}|B$ is a Weak Dutch Book gain for $\underline{P}$ if

$$\sup(\underline{G}|B) = 0. \tag{1}$$

What is a Weak Dutch Book gain?

Let $\underline{P} : \mathcal{D} \to \mathbb{R}$ be a convex (W-coherent) lower prevision. Any admissible gain $\underline{G}|B$ for $\underline{P}$ satisfies

$$\sup(\underline{G}|B) \geq 0.$$

Weak Dutch Book gain

$\underline{G}|B$ is a Weak Dutch Book gain for $\underline{P}$ if

$$\sup(\underline{G}|B) = 0. \tag{1}$$

If $\underline{P}$ is unconditional, (1) simplifies into $\sup \underline{G} = 0$.

What is a Weak Dutch Book gain?

Let $\underline{P} : \mathcal{D} \rightarrow \mathbb{R}$ be a convex (W-coherent) lower prevision. Any admissible gain $\underline{G}|B$ for $\underline{P}$ satisfies

$$\sup(\underline{G}|B) \geq 0.$$

Weak Dutch Book gain

$\underline{G}|B$ is a Weak Dutch Book gain for $\underline{P}$ if

$$\sup(\underline{G}|B) = 0. \tag{1}$$

If $\underline{P}$ is unconditional, (1) simplifies into $\sup \underline{G} = 0$.

Remark: WDBs arise with *extreme probability* events, but *not only*. (*Proposition 3.1* in *Corsato, Pelessoni, Vicig, 2017*)

Local precision for convexity

Proposition

(a) If $\underline{P} : \mathcal{D} \to \mathbb{R}$ is a conditional convex lower prevision and $\underline{G}|B$ is a WDB gain for $\underline{P}$,

Local precision for convexity

Proposition

(a) If $\underline{P} : \mathcal{D} \to \mathbb{R}$ is a conditional convex lower prevision and $\underline{G}|B$ is a WDB gain for $\underline{P}$, then $\exists P$, dF-coherent prevision, $\alpha_P \in \mathbb{R}$ s.t., for $i = 0$ and $\forall\, i = 1, \ldots, n$ with $s_i > 0$,

- either $P(B_i|B) = 0$,
- or $\underline{P}(X_i|B_i) = P(X_i|B_i) + \dfrac{\alpha_P}{P(B_i|B)}$.

Local precision for convexity

Proposition

(*a*) If $\underline{P} : \mathcal{D} \to \mathbb{R}$ is a conditional convex lower prevision and $\underline{G}|B$ is a WDB gain for $\underline{P}$, then $\exists P$, dF-coherent prevision, $\alpha_P \in \mathbb{R}$ s.t., for $i = 0$ and $\forall\, i = 1, \ldots, n$ with $s_i > 0$,

- either $P(B_i|B) = 0$,
- or $\underline{P}(X_i|B_i) = P(X_i|B_i) + \dfrac{\alpha_P}{P(B_i|B)}$.

(*b*) If $\underline{P}$ is unconditional, then, $\forall\, i = 0, 1, \ldots, n$ with $s_i > 0$,

$$\underline{P}(X_i) = P(X_i) + \alpha_P.$$

$\Longrightarrow$ $\underline{P}$ is a 'local' translation of a precise prevision.

Remark:

- in the unconditional case, $\underline{P}(B_i|B) = \underline{P}(\Omega|\Omega) = 1$.

Local precision for W-coherence

Proposition

If $\underline{P} : \mathcal{D} \to \mathbb{R}$ is a conditional W-coherent lower prevision and

Local precision for W-coherence

Proposition

If $\underline{P} : \mathcal{D} \to \mathbb{R}$ is a conditional W-coherent lower prevision and

- $\underline{G}|B$ WDB gain for $\underline{P}$,
- $\mathcal{D}^{+}_{\underline{G}} = \{X_0|B_0\} \cup \{X_i|B_i : s_i \underline{P}(B_i|B) > 0, \text{ for } i = 1, \ldots, n\}$,

Local precision for W-coherence

Proposition

If $\underline{P} : \mathcal{D} \to \mathbb{R}$ is a conditional W-coherent lower prevision and

- $\underline{G}|B$ WDB gain for $\underline{P}$,
- $\mathcal{D}^+_{\underline{G}} = \{X_0|B_0\} \cup \{X_i|B_i : s_i \underline{P}(B_i|B) > 0, \text{ for } i = 1, \ldots, n\}$,

then $\underline{P}$ is a dF-coherent prevision on $\mathcal{D}^+_{\underline{G}}$.

Local precision for W-coherence

Proposition

If $\underline{P} : \mathcal{D} \to \mathbb{R}$ is a conditional W-coherent lower prevision and

- $\underline{G}|B$ WDB gain for $\underline{P}$,
- $\mathcal{D}^+_{\underline{G}} = \{X_0|B_0\} \cup \{X_i|B_i : s_i\underline{P}(B_i|B) > 0,\ \text{for } i = 1, \ldots, n\}$,

then $\underline{P}$ is a dF-coherent prevision on $\mathcal{D}^+_{\underline{G}}$.

Remark:

- specialises the convex case with $\alpha_P = 0$.

WDBs and positive probability events - 1

Proposition

If $\underline{P} : \mathcal{D} \rightarrow \mathbb{R}$ is a conditional W-coherent lower prevision and

- $\underline{G}|B$ WDB gain for $\underline{P}$,
- $E|B \in \mathcal{D}$ and $\underline{P}(E|B) > 0$,

then

$$\sup(\underline{G}|B \wedge E) = 0.$$

WDBs and positive probability events - 1

Proposition

If $\underline{P} : \mathcal{D} \to \mathbb{R}$ is a conditional W-coherent lower prevision and

- $\underline{G}|B$ WDB gain for $\underline{P}$,
- $E|B \in \mathcal{D}$ and $\underline{P}(E|B) > 0$,

then

$$\sup(\underline{G}|B \wedge E) = 0.$$

Special case:

- $\underline{P}$ unconditional, $E = \omega \in \mathbb{P}$, domain of $\underline{G}$. Then

$$\sup \underline{G} = \max \underline{G} = \underline{G}(\omega) = 0.$$

WDBs and positive probability events - 2

Define

$$\mathcal{P} = \{\omega \in \mathbb{P} : \underline{P}(\omega) > 0\}, \quad \mathcal{N} = \{\omega \in \mathbb{P} : \underline{G}(\omega) = 0\}.$$

WDBs and positive probability events - 2

Define

$$\mathcal{P} = \{\omega \in \mathbb{P} : \underline{P}(\omega) > 0\}, \quad \mathcal{N} = \{\omega \in \mathbb{P} : \underline{G}(\omega) = 0\}.$$

$\underline{G}$ WDB gain $\longrightarrow \mathcal{P} \subseteq \mathcal{N}$,

WDBs and positive probability events - 2

Define

$$\mathcal{P} = \{\omega \in \mathbb{P} : \underline{P}(\omega) > 0\}, \quad \mathcal{N} = \{\omega \in \mathbb{P} : \underline{G}(\omega) = 0\}.$$

$$\underline{G} \text{ WDB gain} \longrightarrow \mathcal{P} \subseteq \mathcal{N},$$

$$\underline{G} \text{ WDB gain} \not\longleftarrow \mathcal{P} \subseteq \mathcal{N}.$$

WDBs and positive probability events - 2

Define

$$\mathcal{P} = \{\omega \in \mathbb{P} : \underline{P}(\omega) > 0\}, \quad \mathcal{N} = \{\omega \in \mathbb{P} : \underline{G}(\omega) = 0\}.$$

$$\underline{G} \text{ WDB gain} \longrightarrow \mathcal{P} \subseteq \mathcal{N},$$

$$\underline{G} \text{ WDB gain} \not\longleftarrow \mathcal{P} \subseteq \mathcal{N}.$$

Remarks:

- the relationship between $\mathcal{P}$ and $\mathcal{N}$ may also depend on the stakes s_i; (*Example 4.4* in *Corsato, Pelessoni, Vicig, 2017*)
- a simple test for checking WDBs:

$$\exists\, \omega \in \mathbb{P} : \underline{P}(\omega) > 0,\ \underline{G}(\omega) \neq 0 \longrightarrow \underline{G} \text{ is no a WDB gain.}$$

Vulnerability to Dutch Books with coherence

Question

Which are the agent's beliefs about suffering from real losses under *coherence* assumptions?

Vulnerability to Dutch Books with coherence

Question

Which are the agent's beliefs about suffering from real losses under *coherence* assumptions?

P unconditional dF-coherent probability,
G WDB gain for P.

Vulnerability to Dutch Books with coherence

Question

Which are the agent's beliefs about suffering from real losses under *coherence* assumptions?

P unconditional dF-coherent probability,
G WDB gain for P.

$P(G < 0) = 0.$

(*Crisma, 2006*)

Vulnerability to Dutch Books with coherence

Question

Which are the agent's beliefs about suffering from real losses under *coherence* assumptions?

P unconditional dF-coherent probability,
G WDB gain for P.

$P(G < 0) = 0.$

(*Crisma, 2006*)

$\underline{P}$ unconditional W-coherent lower prevision,
$\underline{G}$ WDB gain for $\underline{P}$.

Vulnerability to Dutch Books with coherence

Question

Which are the agent's beliefs about suffering from real losses under *coherence* assumptions?

P unconditional dF-coherent probability,
G WDB gain for P.

$P(G < 0) = 0.$

(*Crisma, 2006*)

$\underline{P}$ unconditional W-coherent lower prevision,
$\underline{G}$ WDB gain for $\underline{P}$.

$\forall \varepsilon > 0, \underline{P}(\underline{G} \leq -\varepsilon) = 0.$

(*Vicig, 2010*)

Vulnerability to Dutch Books with coherence

Question

Which are the agent's beliefs about suffering from real losses under *coherence* assumptions?

P unconditional dF-coherent probability,
G WDB gain for P.

$P(G < 0) = 0.$

(*Crisma, 2006*)

$\underline{P}$ unconditional W-coherent lower prevision,
$\underline{G}$ WDB gain for $\underline{P}$.

$\forall \varepsilon > 0,\ \underline{P}(\underline{G} \leq -\varepsilon) = 0.$

(*Vicig, 2010*)

General answer

$\underline{P}$ conditional W-coherent lower prevision,
$\underline{G}|B$ WDB gain for $\underline{P}$.

Vulnerability to Dutch Books with coherence

Question

Which are the agent's beliefs about suffering from real losses under *coherence* assumptions?

P unconditional dF-coherent probability,
G WDB gain for P.

$P(G < 0) = 0.$

(*Crisma, 2006*)

$\underline{P}$ unconditional W-coherent lower prevision,
$\underline{G}$ WDB gain for $\underline{P}$.

$\forall \varepsilon > 0,\ \underline{P}(\underline{G} \leq -\varepsilon) = 0.$

(*Vicig, 2010*)

General answer

$\underline{P}$ conditional W-coherent lower prevision,
$\underline{G}|B$ WDB gain for $\underline{P}$.

$\forall \varepsilon > 0,\ \underline{P}(\underline{G}|B \leq -\varepsilon) = 0.$

Vulnerability to Dutch Books (continued)

$\underline{P}$ conditional W-coherent lower prevision,
$\underline{G}|B$ WDB gain for $\underline{P}$.

$\forall \varepsilon > 0$, $\underline{P}(\underline{G}|B \leq -\varepsilon) = 0$.

Vulnerability to Dutch Books (continued)

$\underline{P}$ conditional W-coherent lower prevision,
$\underline{G}|B$ WDB gain for $\underline{P}$.

$\forall \varepsilon > 0$, $\underline{P}(\underline{G}|B \leq -\varepsilon) = 0$.

Anyway,

- under W-coherence, for some $\varepsilon > 0$,

$$\overline{P}(\underline{G}|B \leq -\varepsilon) \text{ may be } > 0 \quad (\text{even } = 1);$$

Vulnerability to Dutch Books (continued)

$\underline{P}$ conditional W-coherent lower prevision,
$\underline{G}|B$ WDB gain for $\underline{P}$.

$$\forall \varepsilon > 0,\ \underline{P}(\underline{G}|B \leq -\varepsilon) = 0.$$

Anyway,

- under W-coherence, for some $\varepsilon > 0$,

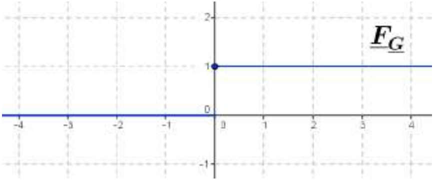
$$\overline{P}(\underline{G}|B \leq -\varepsilon) \text{ may be } > 0 \quad (\text{even } = 1);$$

- under convexity, for some $\varepsilon > 0$,

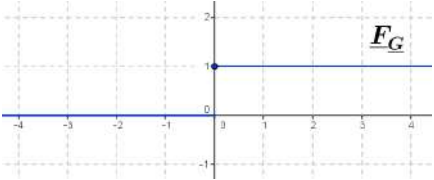
$$\underline{P}(\underline{G}|B \leq -\varepsilon) \text{ may be } > 0.$$

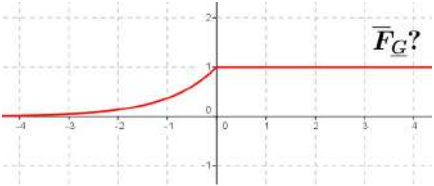
(*Examples 5.1 and 5.2* in *Corsato, Pelessoni, Vicig, 2017*)

Uncertainty in the p-box for a coherent WDB gain

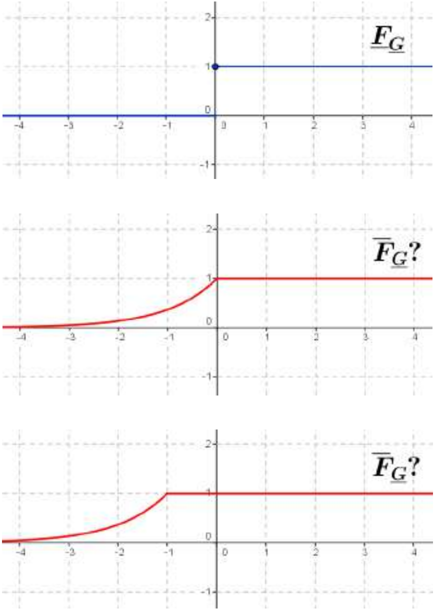


Uncertainty in the *p*-box for a coherent WDB gain

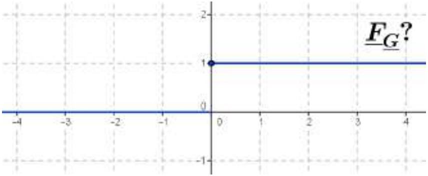


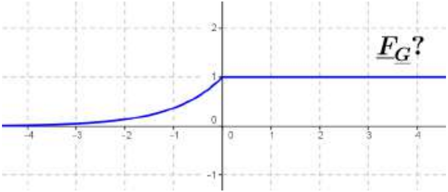


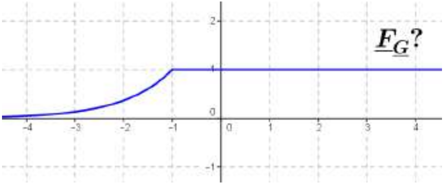
Uncertainty in the p-box for a coherent WDB gain



Uncertainty in the *p*-box for a convex WDB gain







Notions of strict consistency

Problem

Incurring a Weak Dutch Book is not quite *desirable* for an agent.

Notions of strict consistency

Problem

Incurring a Weak Dutch Book is not quite *desirable* for an agent.

(Oldest) solution: strengthen the consistency conditions to rule out WDBs.

Strict consistency

Let $\underline{P} : \mathcal{D} \to \mathbb{R}$ be a convex (W-coherent) lower prevision.
$\underline{P}$ is a strictly convex (strictly W-coherent) lower prevision if, for any admissible gain $\underline{G}|B$ for $\underline{P}$, it holds that

$$\text{either } \underline{G}|B = 0 \quad \text{or } \sup(\underline{G}|B) > 0.$$

Characterisation of strict coherence

$\mathcal{A}$ algebra of unconditional events,

$P : \mathcal{A} \to \mathbb{R}$ dF-coherent.

Characterisation of strict coherence

$\mathcal{A}$ algebra of unconditional events,

$P : \mathcal{A} \to \mathbb{R}$ *dF*-coherent.

P strictly *dF*-coherent

$\updownarrow$

$\forall E \in \mathcal{A} \setminus \{\varnothing\}, \quad P(E) > 0.$

(*Kemeny, 1955; Shimony, 1955*)

Characterisation of strict coherence

$\mathcal{A}$ algebra of unconditional events,

$P : \mathcal{A} \to \mathbb{R}$ dF-coherent.

P strictly dF-coherent

$\updownarrow$

$\forall E \in \mathcal{A} \setminus \{\varnothing\}, \quad P(E) > 0.$

(*Kemeny, 1955; Shimony, 1955*)

$\mathcal{D}$ set of conditional gambles,

$\forall$ WDB gain $\underline{G}|B \neq 0$,

$\exists \varepsilon > 0 : (\underline{G}|B \leq -\varepsilon) \in \mathcal{D}$, non-impossible,

$\underline{P} : \mathcal{D} \to \mathbb{R}$ W-coherent.

Characterisation of strict coherence

$\mathcal{A}$ algebra of unconditional events,

$P : \mathcal{A} \to \mathbb{R}$ dF-coherent.

P strictly dF-coherent

$\updownarrow$

$\forall E \in \mathcal{A} \setminus \{\varnothing\}, \quad P(E) > 0.$

(*Kemeny, 1955; Shimony, 1955*)

$\mathcal{D}$ set of conditional gambles,

$\forall$ WDB gain $\underline{G}|B \neq 0$,

$\exists \varepsilon > 0 : (\underline{G}|B \leq -\varepsilon) \in \mathcal{D}$, non-impossible,

$\underline{P} : \mathcal{D} \to \mathbb{R}$ W-coherent.

$\underline{P}$ strictly W-coherent

$\updownarrow$

$\forall E|B \in \mathcal{D} \setminus \{\varnothing|B\}, \quad \underline{P}(E|B) > 0.$

Characterisation of strict coherence

$\mathcal{A}$ algebra of unconditional events,

$P : \mathcal{A} \to \mathbb{R}$ dF-coherent.

P strictly dF-coherent

$\updownarrow$

$\forall E \in \mathcal{A} \setminus \{\varnothing\}, \quad P(E) > 0.$

(*Kemeny, 1955; Shimony, 1955*)

$\mathcal{D}$ set of conditional gambles,

$\forall$ WDB gain $\underline{G}|B \neq 0$,

$\exists \varepsilon > 0 : (\underline{G}|B \leq -\varepsilon) \in \mathcal{D}$, non-impossible,

$\underline{P} : \mathcal{D} \to \mathbb{R}$ W-coherent.

$\underline{P}$ strictly W-coherent

$\updownarrow$

$\forall E|B \in \mathcal{D} \setminus \{\varnothing|B\}, \quad \underline{P}(E|B) > 0.$

Special case:

$\mathbb{P}$ partition of Ω,

$\mathcal{D} = \mathcal{L}(\mathbb{P})$ linear space of gambles on $\mathbb{P}$.

Characterisation of strict coherence

$\mathcal{A}$ algebra of unconditional events,

$P : \mathcal{A} \to \mathbb{R}$ dF-coherent.

P strictly dF-coherent

$\updownarrow$

$\forall E \in \mathcal{A} \setminus \{\varnothing\}, \quad P(E) > 0.$

(*Kemeny, 1955; Shimony, 1955*)

$\mathcal{D}$ set of conditional gambles,

$\forall$ WDB gain $\underline{G}|B \neq 0$,

$\exists \varepsilon > 0 : (\underline{G}|B \leq -\varepsilon) \in \mathcal{D}$, non-impossible,

$\underline{P} : \mathcal{D} \to \mathbb{R}$ W-coherent.

$\underline{P}$ strictly W-coherent

$\updownarrow$

$\forall E|B \in \mathcal{D} \setminus \{\varnothing|B\}, \quad \underline{P}(E|B) > 0.$

Special case:

$\mathbb{P}$ partition of Ω,

$\mathcal{D} = \mathcal{L}(\mathbb{P})$ linear space of gambles on $\mathbb{P}$.

$\underline{P}$ strictly W-coherent

$\updownarrow$

- $\underline{P}$ W-coherent,
- $\forall E \in \mathcal{L}(\mathbb{P}) \setminus \{\varnothing\}, \quad \underline{P}(E) > 0.$

Strict consistency vs other approaches

- Strict coherence is essentially confined to a countable environment, even with W-coherence.

Strict consistency vs other approaches

- Strict coherence is essentially confined to a countable environment, even with W-coherence.
- Alternative approaches to hedging WDBs: via
  - desirability,
    (*Williams, 1975; Quaeghebeur, de Cooman, Hermans, 2015*)
  - buying/selling schemes,
    (*Walley, 1991; Wagner, 2007*)
  - a qualitative model.
    (*Pedersen, 2014*)

We did not mention here

- Further properties for strict consistency have been obtained.

We did not mention here

- Further properties for strict consistency have been obtained.
- Some propositions describing WDBs derive from more general results.

Conclusions

- ✔ WDBs are extreme situations to cope with.

Conclusions

- ✔ WDBs are extreme situations to cope with.
- ✔ WDBs induce local precision properties with W-coherent or convex lower previsions.

Conclusions

- ✔ WDBs are extreme situations to cope with.
- ✔ WDBs induce local precision properties with W-coherent or convex lower previsions.
- ✔ The agent's confidence to avoid a proper Dutch Book may decrease with weaker consistency notions.

See you...

...at the Poster Session

Thank you for your attention!