Agreeing to Disagree and Dilation

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Dilation is Responsible for "Agreeing to Disagree"

- Aumann (1976) showed that it is impossible for two Bayesian agents with a common precise prior to "agree to disagree".
- With a common imprecise prior, "agreeing to disagree" is possible, but only thanks to the phenomenon of dilation.

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- All these are *commonly* known: each agent knows them, knows that each knows them, knows that each knows that each knows them, … *ad infinitum*.

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- Then, if it is common knowledge that agent 1's posterior of an event H is p_1 and agent 2's posterior of H is p_2, then $p_1 = p_2$.

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- Then, if it is common knowledge that agent 1's posterior of an event H is p_1 and agent 2's posterior of H is p_2, then $p_1 = p_2$.
- That is, the agents cannot agree to disagree!

Example of Agreement

$\mathcal{P}^2$

$\mathcal{P}^1$		
	w_1	w_2
	w_3	w_4

$$\mathcal{P}^1 = \{\{w_1, w_2\}, \{w_3, w_4\}\}; \mathcal{P}^2 = \{\{w_1, w_3\}, \{w_2, w_4\}\}; H = \{w_1, w_4\}.$$

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- Suppose $p = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$, and suppose w_1 is the true state, at which agent 1 learns $\{w_1, w_2\}$, and agent 2 learns $\{w_1, w_3\}$.

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- So, $p_1(H) = p(H|\{w_1, w_2\}) = \frac{1}{2} = p(H|\{w_1, w_3\}) = p_2(H)$.

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- Note that it is common knowledge that $p_1(H) = \frac{1}{2}$, for agent 2 can see (and agent 1 can see that agent 2 can see) that no matter which state is true, agent 1's posterior of H would be $\frac{1}{2}$.

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- Similarly, it is also common knowledge that $p_2(H) = \frac{1}{2}$.

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- But this is *not* agreeing to disagree, for neither agent knows the other's posterior of H'.

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- But this is *not* agreeing to disagree, for neither agent knows the other's posterior of H'.
- Indeed, if agent 1 makes her posterior known to agent 2, agent 2 will update his and reach an agreement with agent 1.
- In general, if agents keep communicating credences to each other and updating accordingly, then the posteriors at the equilibrium must be equal (Geanakoplos & Polemarchakis, 1982).

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- That is, two Bayesian agents learning different pieces of evidence can have different posterior sets that are common knowledge, or even different lower or upper posteriors that are common knowledge.
- The possibility of agreeing to disagree on the lower or upper posterior is due *solely* to the possibility of dilation.

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- Instead of a precise prior, suppose the two agents begin with a common imprecise prior, an ϵ-contaminated class: $\mathbf{Q} = \{0.8p + 0.2q \mid q \in \Lambda\}$, where $p = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$, and Λ is the set of all distributions over $\mathcal{A}$.

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- Suppose the true state is w_1, at which $\mathbf{Q}_1(H) = \mathbf{Q}(H|\{w_1, w_2\}) = [\frac{1}{3}, \frac{2}{3}]$, whereas $\mathbf{Q}_2(H) = \mathbf{Q}(H|\Omega) = [\frac{2}{5}, \frac{3}{5}]$.

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- Even though it is common knowledge that $\mathbf{Q}_1(H) = [\frac{1}{3}, \frac{2}{3}]$ and that $\mathbf{Q}_2(H) = [\frac{2}{5}, \frac{3}{5}]$!

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Dilation in the Example

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- Observe that H is *dilated* by $\mathcal{P}^1$, in the sense that for every $E \in \mathcal{P}^1$, $[\underline{\mathbf{Q}}(H|E), \overline{\mathbf{Q}}(H|E)] = [\frac{1}{3}, \frac{2}{3}]$ strictly contains $[\underline{\mathbf{Q}}(H), \overline{\mathbf{Q}}(H)] = [\frac{2}{5}, \frac{3}{5}]$.

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Take-home Message

It is no accident that dilation occurs when the agents can agree to disagree!

Some Notations

- Suppose two agents have the same (imprecise) prior, $\mathbf{Q}$, over a measurable space $(\Omega, \mathcal{A})$.
- Agent i learns (privately) the value of a (measurable) partition of Ω, $\mathcal{P}_i$, and updates by full Bayesian conditioning.
- Assume that every member in the coarsest common refinement of $\mathcal{P}^1$ and $\mathcal{P}^2$ is non-null under every measure in $\mathbf{Q}$.
- Let w denote the true state of the world, and $\mathcal{P}^i(w)$ the member of $\mathcal{P}^i$ that contains w. Agent i's posterior of H is $\mathbf{Q}(H|\mathcal{P}^i(w))$.
- $\underline{\mathbf{Q}}(H|E) = \inf_{p \in \mathbf{Q}} p(H|E)$; $\overline{\mathbf{Q}}(H|E) = \sup_{p \in \mathbf{Q}} p(H|E)$.
- Let $\mathcal{P}$ be the finest common coarsening of $\mathcal{P}^1$ and $\mathcal{P}^2$. Let $\mathcal{C}_0 = \mathcal{P}(w)$, which is the finest event that is common knowledge.
- Let $\mathcal{P}^i_0 = \{E \in \mathcal{P}^i \mid E \cap \mathcal{C}_0 \neq \emptyset\}$. Obviously $\mathcal{P}^i_0$ is a partition of $\mathcal{C}_0$.

Dilation is Necessary for Agreeing to Disagree

Definition (Dilation)

$\mathcal{P}_0^i$ is said to dilate H (with respect to $\mathbf{Q}$) if for every $E \in \mathcal{P}_0^i$, the closed interval $[\underline{\mathbf{Q}}(H|E), \overline{\mathbf{Q}}(H|E)]$ strictly contains $[\underline{\mathbf{Q}}(H|\mathcal{C}_0), \overline{\mathbf{Q}}(H|\mathcal{C}_0)]$.

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Theorem (Agreement on lower and upper probabilities)

Suppose for both $i = 1, 2$, $\mathcal{P}_0^i$ does not dilate H. If both $\mathbf{Q}(H|\mathcal{P}^1(w))$ and $\mathbf{Q}(H|\mathcal{P}^2(w))$ are common knowledge, then $\underline{\mathbf{Q}}(H|\mathcal{P}^1(w)) = \underline{\mathbf{Q}}(H|\mathcal{P}^2(w))$ and $\overline{\mathbf{Q}}(H|\mathcal{P}^1(w)) = \overline{\mathbf{Q}}(H|\mathcal{P}^2(w))$.

A Result on Full Agreement

Theorem (Full agreement)

Suppose $\mathbf{Q}$ is closed and connected, and for both $i = 1, 2$, $\mathcal{P}_0^i$ does not dilate H. If both $\mathbf{Q}(H|\mathcal{P}^1(w))$ and $\mathbf{Q}(H|\mathcal{P}^2(w))$ are common knowledge, then $\mathbf{Q}(H|\mathcal{P}^1(w)) = \mathbf{Q}(H|\mathcal{P}^2(w))$.

A Corollary for Density Ratio Priors

Definition (Density ratio class)

Let $\Omega = \{w_1, ..., w_n\}$ and $\mathcal{A} = \mathbb{P}(\Omega)$. A density ratio class is defined by

$$\mathbf{D}_{p,k} = \{(q_1, ..., q_n) \mid \sum q_j = 1 \quad and \quad \frac{q_h}{q_j} \leq k\frac{p_h}{p_j}, \forall 1 \leq h, j \leq n\}$$

where $k \geq 1$ and $(p_1, ..., p_n)$ is a positive probability vector.

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Corollary (Full agreement for density ratio priors)

Suppose $\mathbf{Q}$ is a density ratio class. If both $\mathbf{Q}(H|\mathcal{P}^1(w))$ and $\mathbf{Q}(H|\mathcal{P}^2(w))$ are common knowledge, then $\mathbf{Q}(H|\mathcal{P}^1(w)) = \mathbf{Q}(H|\mathcal{P}^2(w))$.

Dilation-averse Agents

- Suppose the agents are commonly known to be *dilation-averse*, who adopt $\mathbf{Q}(H|\mathcal{C}_0)$ instead of $\mathbf{Q}(H|\mathcal{P}^i(w))$ as their posterior if $\mathcal{P}_0^i$ dilates H, and otherwise update by Bayesain conditioning.

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- Then they cannot agree to disagree on the lower or upper posterior.
- If the (common) prior is closed and connected, they cannot agree not to fully agree.

Conclusion and Further Questions

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- Under the usual topological assumptions, the presence of dilation is necessary for agreeing not to fully agree.
- All the results can be generalized to Geanakoplos & Polemarchakis's communication setting.
- What about agents whose priors agree only *partially*?
- What about agents who update by other rules, e.g., the Dempster-Shafer rule?