RELEVANT PATH SEPARATION:
A FASTER METHOD FOR TESTING INDEPENDENCIES IN BAYESIAN NETWORKS

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OUTLINE

- D-SEPARATION
- MOTIVATION
- RP-SEPARATION
- EXPERIMENTAL RESULTS
- CONCLUSION

Introduction

- d-separation was instrumental in the founding of Bayesian networks

- it continues to be useful today
  - causal inference in statistics (Pearl, 2016)
  - cause and correlation in biology (Shipley, 2016)
  - population extrapolation (Pearl and Bareinboim, 2014)
  - handling missing data (Mohan and Pearl, 2014)
  - deep learning (Goodfellow et al., 2016)

Introduction

- d-separation is a graphical method for testing independencies in BNs

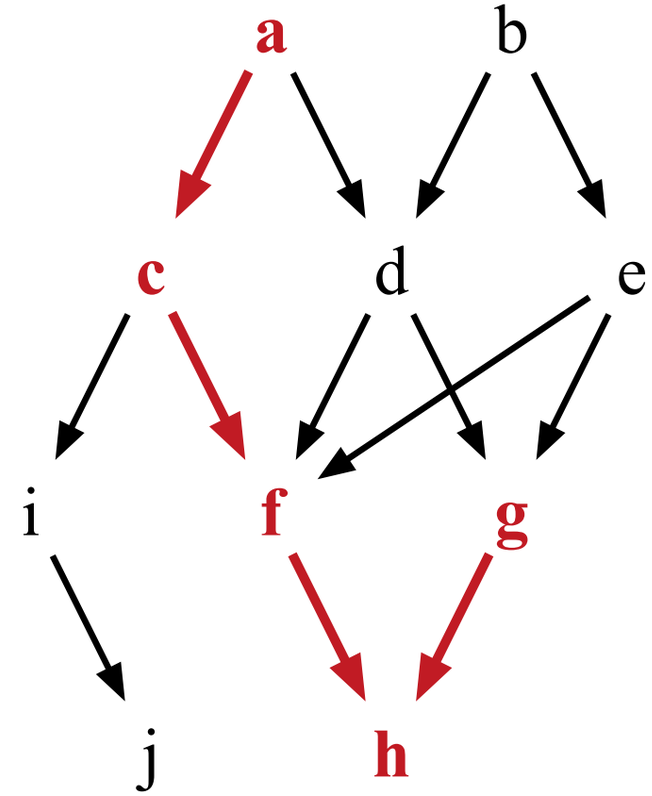
- $\mathbf{I(X,Y,Z)}$ means $\mathbf{X}$ and $\mathbf{Z}$ are conditionally independent given $\mathbf{Y}$

The Key Question

- determine whether there exists an active path from $\mathbf{X}$ to $\mathbf{Z}$ with respect to $\mathbf{Y}$
- when explaining the test of independence $\mathbf{I}(\mathbf{X}, \mathbf{Y}, \mathbf{Z})$, the probabilistic reasoning literature tends to discuss traversing irrelevant paths

Example: Irrelevant Path

I(a,e,g)



- Is the path (a,c), (c,f), (f,h), (h,g) active or blocked?
- This path is **necessarily blocked**

Practical Implementations

- Geiger et al. (1989)
- Bayes-Ball (BB) (Shachter, 1998)
- REACHABLE algorithm (Koller and Friedman, 2009)
- also explore irrelevant paths

GEIGER ET AL. (1989)

- first linear implementation

- explores every edge in the BN

BAYES-BALL AND REACHABLE

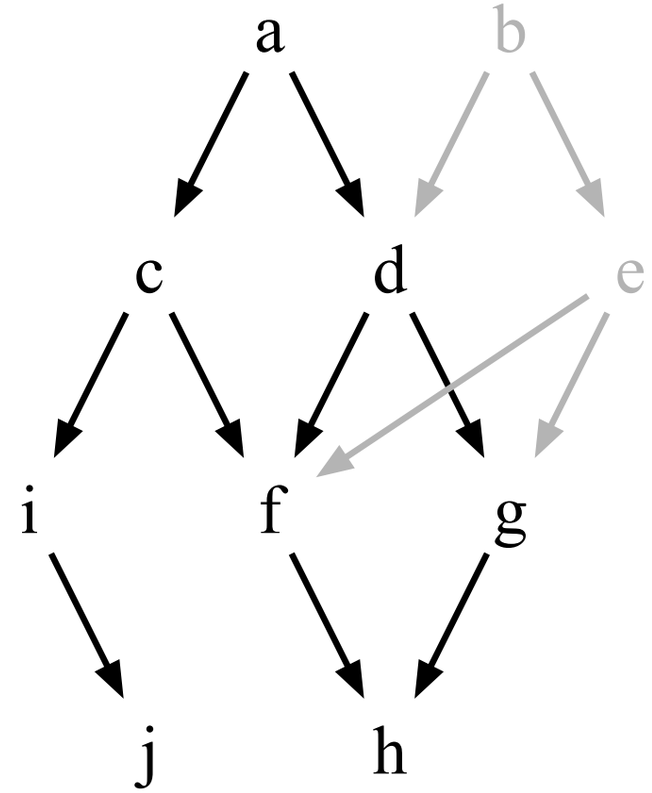
- Bayes-Ball was the first method to explore the **active** part of a BN

- explore all nodes reachable along active paths from $\mathbf{X}$ with respect to $\mathbf{Y}$

- REACHABLE tests $\mathbf{I(X,Y,Z)}$ this way

The Active Part of a BN

I(a,e,g)



An Overlooked Result

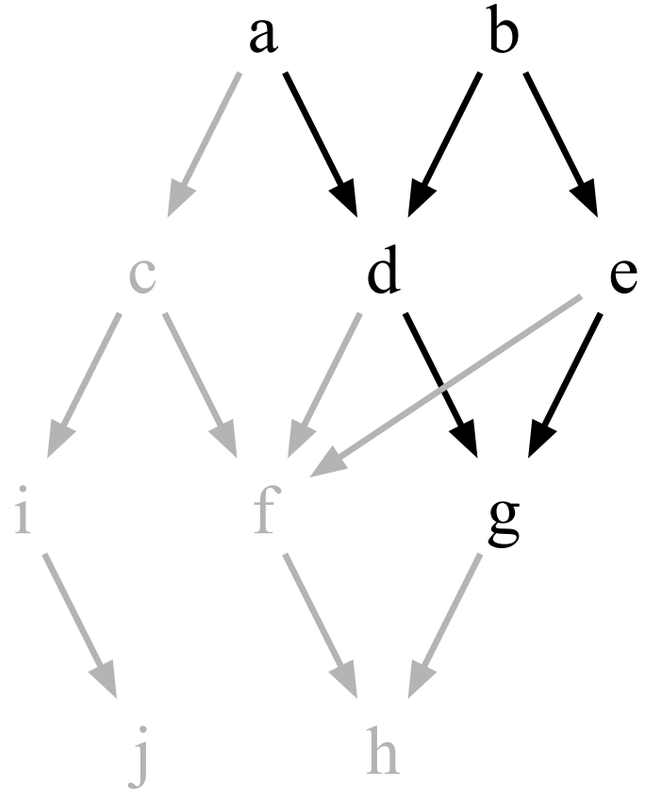
- For $\mathbf{I(X,Y,Z)}$, Lauritzen et al. (1990) established that all active paths from $\mathbf{X}$ to $\mathbf{Z}$ given $\mathbf{Y}$ can only involve variables in

$$\mathbf{XYZ} \cup \mathbf{An(XYZ)}$$

- we call this the **relevant part** of a BN

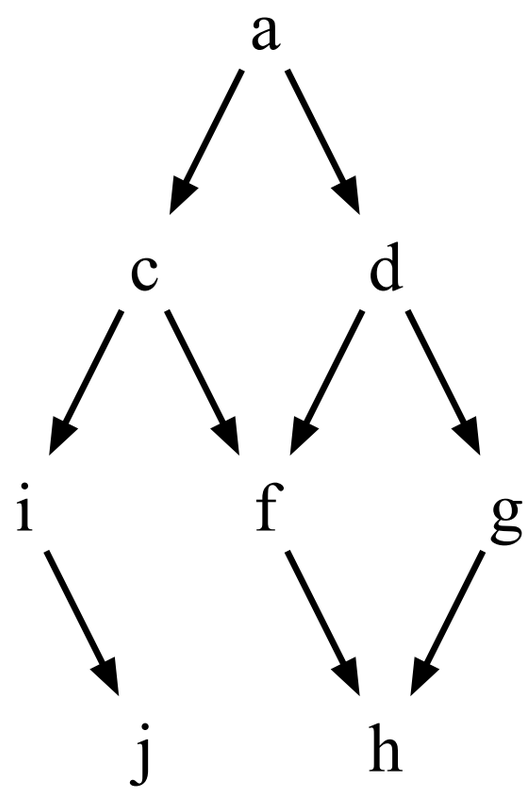
The Relevant Part of a BN

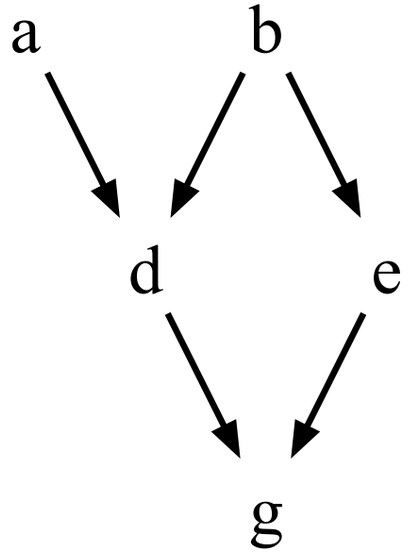
I(a,e,g)



Active Versus relevant

I(a,e,g)





Relevant Path Separation

- a path is **relevant**, if it only involves nodes in

$$XYZ \cup An(XYZ)$$

- restrict traversal to the **intersection** of the active and relevant parts of a BN

- explore relevant paths that are active

- linear implementation of RP-REACHABLE
- based upon REACHABLE
- lines 4, 19, 22, 27, 31

Algorithm 1 Given an independence $I(X, Y, Z)$, RP-REACHABLE traverses all active paths from X within the relevant part of a BN $\mathcal{B}$.

```
1:  procedure RP-REACHABLE(X,Y,Z,B)
2:      ▷ Initialization
3:      A ← Y ∪ An(Y)
4:      XYZ^up ← XYZ ∪ An(XYZ)                          ▷ The relevant part of B
5:      for v ∈ X do                                    ▷ (Variable,direction) to be visited
6:          L ← L ∪ {(↑, v)}
7:      V ← ∅                                           ▷ (Variable,direction) marked as visited
8:      R ← ∅                                           ▷ Variables reachable via active path
9:      ▷ Starting from X traverse relevant paths that are active.
10:     while L ≠ ∅ do                                  ▷ While variables to be checked
11:         Select (d, v) in L
12:         L ← L − {(d, v)}
13:         if (d, v) ∉ V then                          ▷ If v has not been visited from direction d
14:             if v ∉ Y then
15:                 R ← R ∪ {v}                         ▷ v is reachable
16:             V ← V ∪ {(d, v)}                        ▷ Mark v as visited from direction d
17:             if d =↑ and v ∉ Y then
18:                 for v_i ∈ Pa(v) do                  ▷ v is open serial
19:                     if v_i ∈ XYZ^up then            ▷ Only explore relevant paths
20:                         L ← L ∪ {(↑, v_i)}
21:                 for v_i ∈ Ch(v) do                  ▷ v is open divergent
22:                     if v_i ∈ XYZ^up then            ▷ Only explore relevant paths
23:                         L ← L ∪ {(↓, v_i)}
24:             else if d =↓ then
25:                 if v ∉ Y then
26:                     for v_i ∈ Ch(v) do              ▷ v is open serial
27:                         if v_i ∈ XYZ^up then        ▷ Only explore relevant paths
28:                             L ← L ∪ {(↓, v_i)}
29:                 if v ∈ A then
30:                     for v_i ∈ Pa(v) do              ▷ v is open convergent
31:                         if v_i ∈ XYZ^up then        ▷ Only explore relevant paths
32:                             L ← L ∪ {(↑, v_i)}
33:     return R
34: end procedure
```

EXPERIMENTAL RESULTS

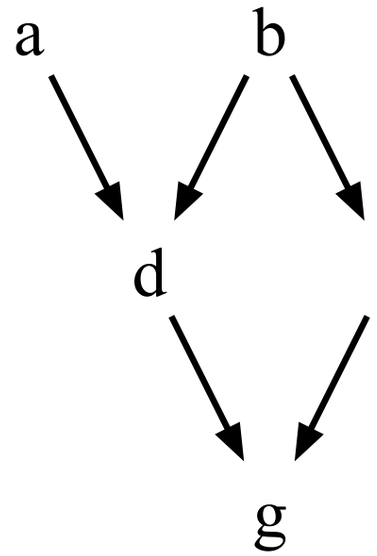
BN	Vars	REACHABLE	RP-REACHABLE	savings over REACHABLE
child	20	1.27E-04	**1.14E-04**	10%
insurance	27	1.93E-04	**1.77E-04**	8%
water	32	1.54E-04	**1.38E-04**	10%
mildew	35	**1.62E-04**	1.70E-04	-5%
alarm	37	**1.39E-04**	1.42E-04	-2%
barley	48	2.51E-04	**2.40E-04**	4%
hailfinder	56	1.70E-04	**1.52E-04**	11%
hepar2	70	3.14E-04	**1.95E-04**	38%
win95pts	76	1.21E-04	**9.90E-05**	18%
pathfinder	109	4.84E-04	**1.19E-04**	75%
munin1	186	5.70E-04	**2.60E-04**	54%
andes	223	8.21E-04	**6.81E-04**	17%
diabetes	413	2.38E-03	**2.23E-03**	6%
pigs	441	3.67E-04	**1.11E-04**	70%
link	724	1.20E-03	**3.70E-04**	69%
munin2	1003	7.66E-04	**1.95E-04**	75%
munin4	1038	1.57E-03	**2.47E-04**	84%
munin	1041	1.53E-03	**2.44E-04**	84%
munin3	1041	1.77E-03	**2.45E-04**	86%
Average Time		6.89E-04	**3.23E-04**	53%

EXPERIMENTAL RESULTS

- RP-REACHABLE was faster than REACHABLE in 17 out of 19 BNs

- RP-REACHABLE was faster than REACHABLE by 53% on average

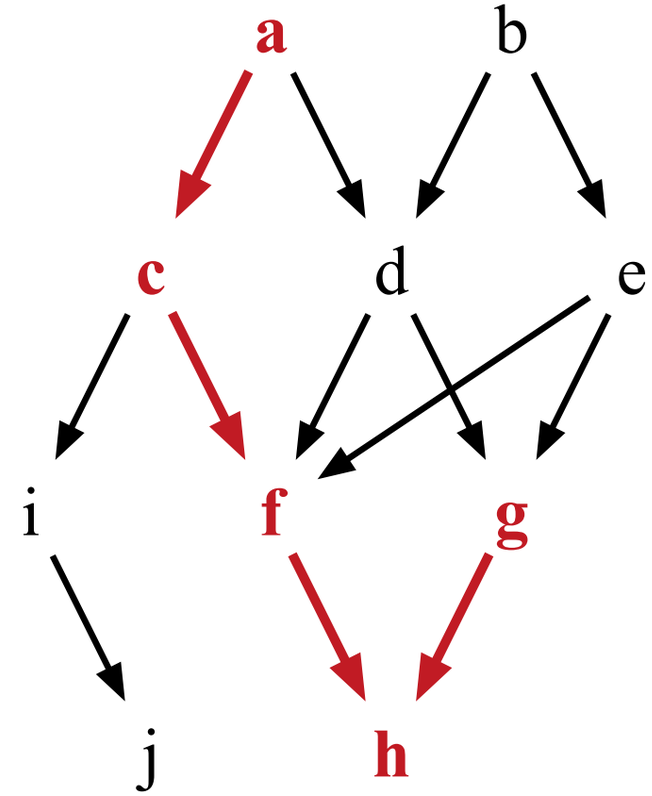
TEACHING D-SEPARATION



- only test **relevant paths**
- the path (a,d), (d,g) is **active**
- the path (a,d), (d,b), (b,e), (e,g) is **blocked**

Example: Irrelevant Path

I(a,e,g)



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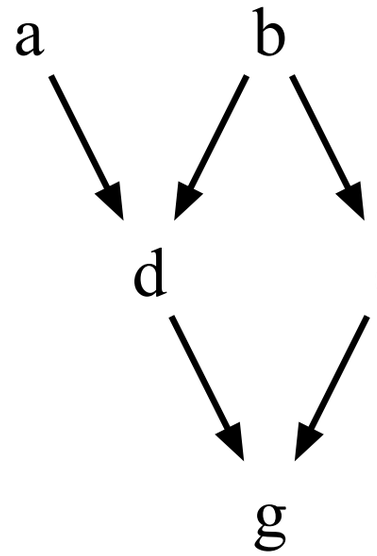

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Conclusion

- rp-separation is a new method for testing independencies in BNs
- considers the intersection of the active and relevant parts of a BN
- faster in 17 out of 19 BNs by 53% on average
- may be useful in teaching d-separation