LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Statistical Matching of Discrete Data by Bayesian Networks

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Statistical matching

A

denomination	gender	age
no rel. community	m	[17.9, 21.9]
Roman Catholic	f	(45, 48.8]
Roman Catholic	m	(45, 48.8]
Roman Catholic	f	(41.1, 45]
Protestant	m	(48.8, 52.6]

⋮

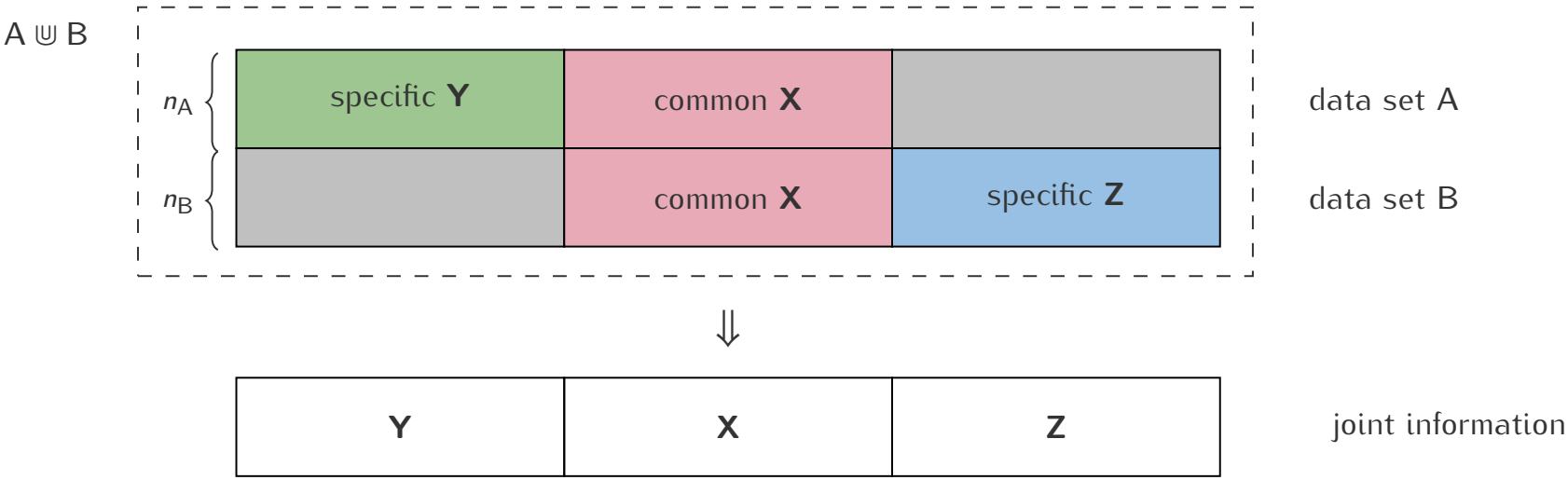
gender	age	alcohol consumption
m	(33.4,37.2]	none
m	(79.6,83.5]	several times/month
m	(33.4,37.2]	several times/month
f	(37.2,41.1]	none

⋮

B

Statistical matching

(e.g. D'Orazio et al. (2006))



- Assumption: conditional independence of **Y** and **Z** given **X**
- Parametric macro approach:

$$L(\mathbf{p}^{\mathrm{A}\uplus\mathrm{B}}|\mathrm{A}\uplus\mathrm{B}) = \prod_{i\in\mathcal{I}_\mathrm{A}} p_{\mathbf{Y}|\mathbf{X}}(\mathbf{y}_i|\mathbf{x}_i) \prod_{i\in\mathcal{I}_\mathrm{B}} p_{\mathbf{Z}|\mathbf{X}}(\mathbf{z}_i|\mathbf{x}_i) \prod_{i\in\mathcal{I}_\mathrm{A}\cup\mathcal{I}_\mathrm{B}} p_{\mathbf{X}}(\mathbf{x}_i)$$

Bayesian networks – basic concepts and notations

(e.g. Koller and Friedman (2009))

- Discrete random variables $\mathbf{W} = (W_1, \ldots, W_s)'$
- Global probability distribution $P(\mathbf{W} = \mathbf{w}) = P(W_1 = w_1, \ldots, W_s = w_s)$
- Directed acyclic graph (DAG) $\mathcal{G}_{\mathbf{W}}$, where
  - each random variable W_m, $m = 1, \ldots, s$, is represented by an eponymous node
  - the dependencies among the random variables are represented by a set of directed edges between pairs of nodes
- *Chain rule* of Bayesian networks

$$P(\mathbf{W} = \mathbf{w}) = \prod_{m=1}^{s} P(W_m = w_m | \mathbf{Pa}(W_m) = \mathbf{pa}(W_m)) =: \prod_{m=1}^{s} p(w_m | \mathbf{pa}(W_m))$$

The German General Social Survey

(GESIS – Leibniz Institute for the Social Sciences (2013))

- Survey from 2012 covers 3480 observations on 752 variables, inter alia, regarding to demography, religiousness and physical health of the respondents
- Idea: pretend that variables measured in the GGSS data came from two different sources
- Extraction of 17 variables for illustration
  - **common**: *sex*, *age*, *graduation*, professional *activity*, *marital* status, and net *income* of the respondents
  - **specific in** A: *denomination*, frequency of *church*goings, frequency of experiencing the presence of *God* through faith, frequency of experiences that can only be explained through the intervention of *supernatural* powers, any experience with miracle *healers*/spirit healers, and frequency of *pray*ing
  - **specific in** B: frequency of visiting a *doctor*, *hospital* stay in the last 12 month, number of *cigarettes* per day, *alcoholic* beverages per day, and general *health*

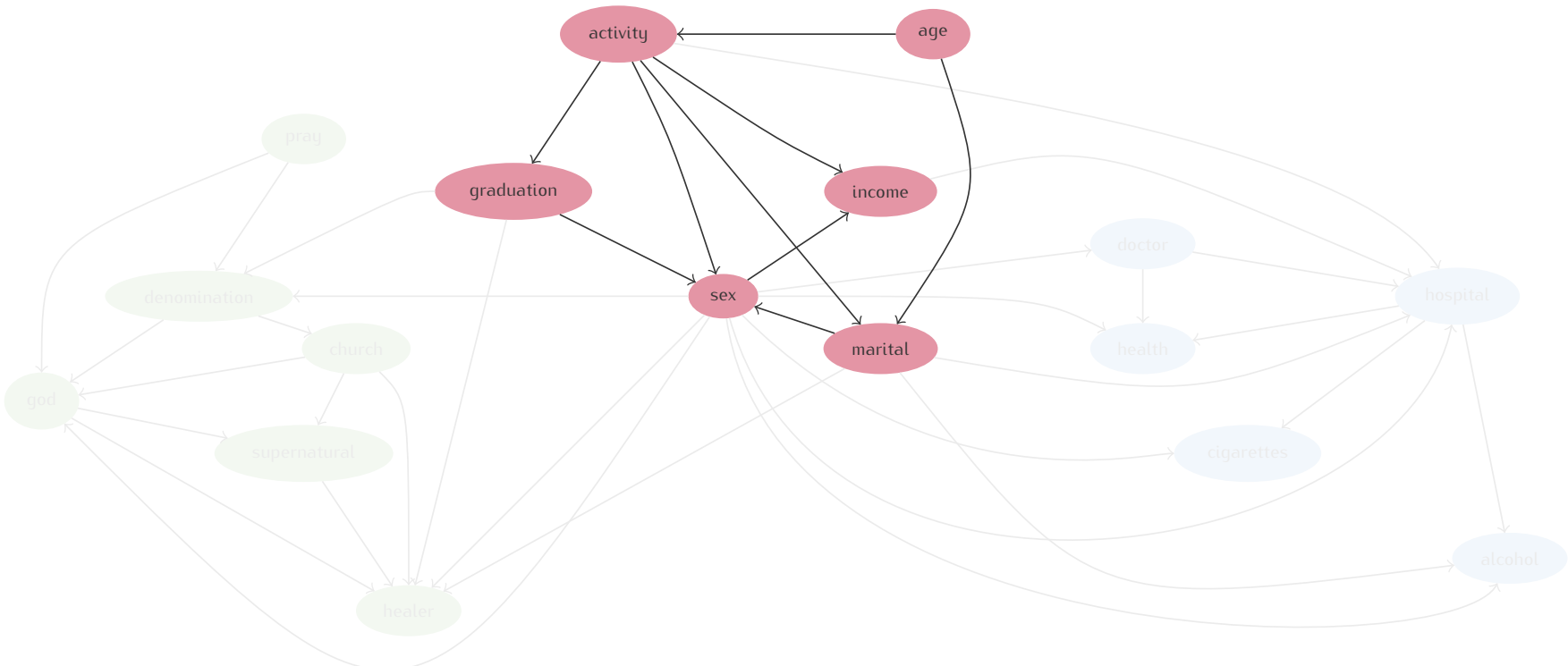
$\Rightarrow$ A and B can be matched as if they came from different surveys

Statistical matching by Bayesian networks

- Representation of $\mathbf{Y} \perp \mathbf{Z} \mid \mathbf{X}$ by Bayesian networks and incorporation of further information on the conditional independence structure
- Restriction of the DAG to basic form $\mathbf{Y} \leftarrow \mathbf{X} \rightarrow \mathbf{Z}$
- Basic procedure:
  1. Estimate two DAGs on A and on B and combine them to a joint DAG
     - 1.1 fix graph structure for the common variables
     - 1.2 individual graph structures for the common variables
  2. Estimate the corresponding local parameters and the global probability distribution
  3. Generate synthetic data set by imputation

Step 1: estimation and combination of the graph structures

fix graph structure for the common variables



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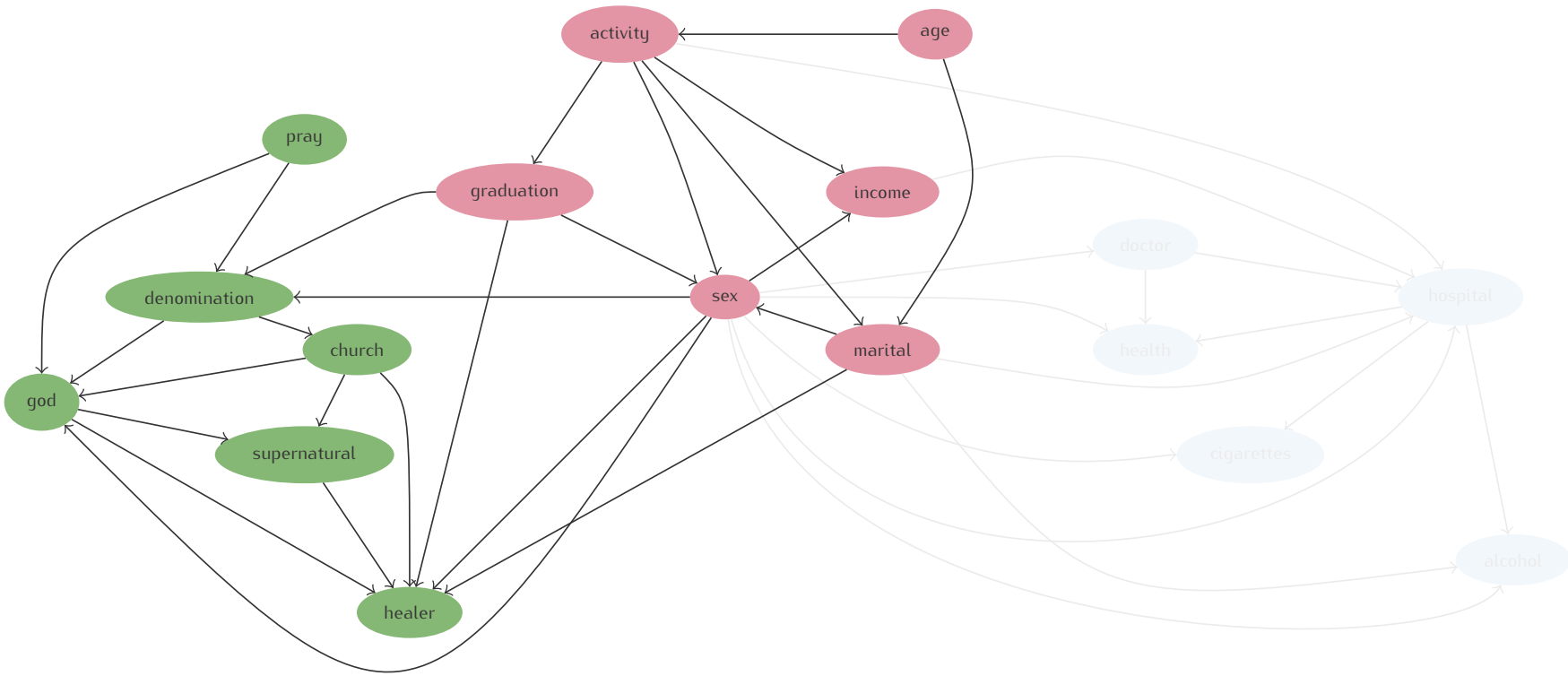

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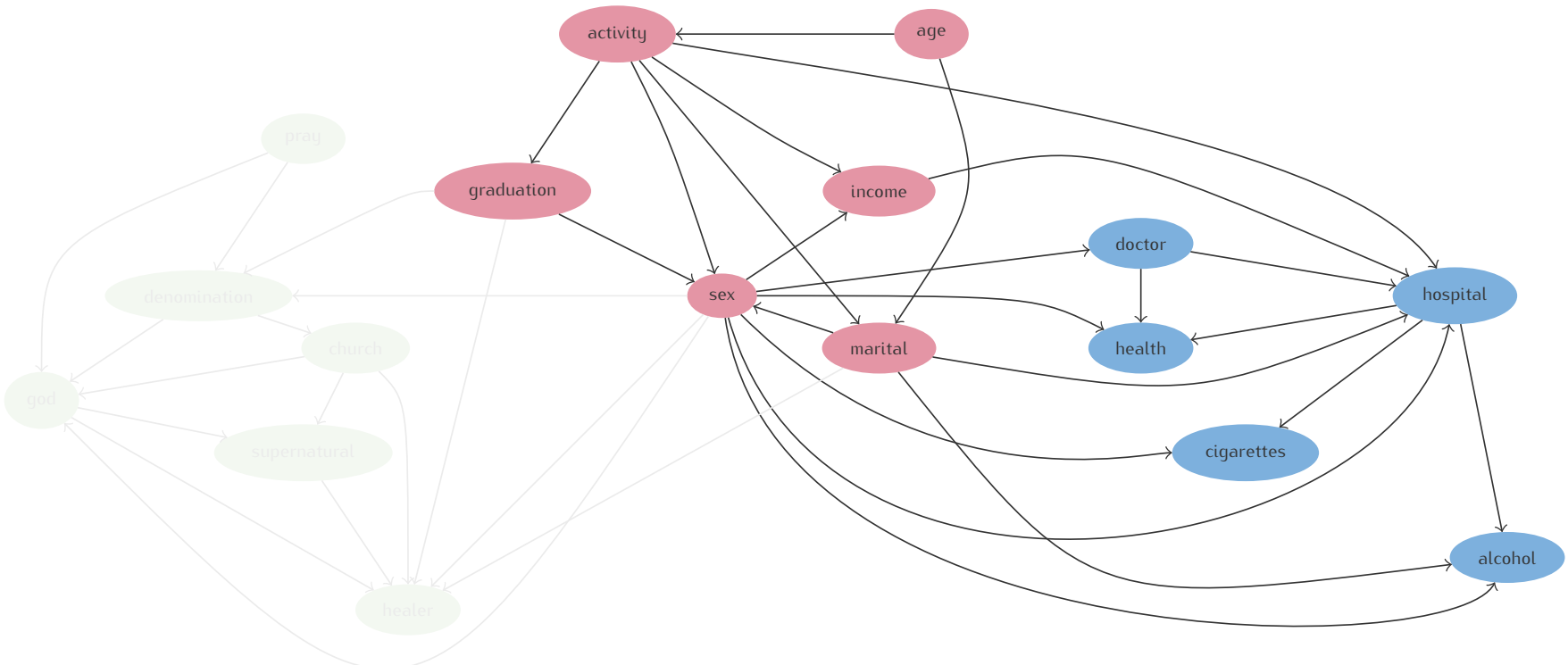

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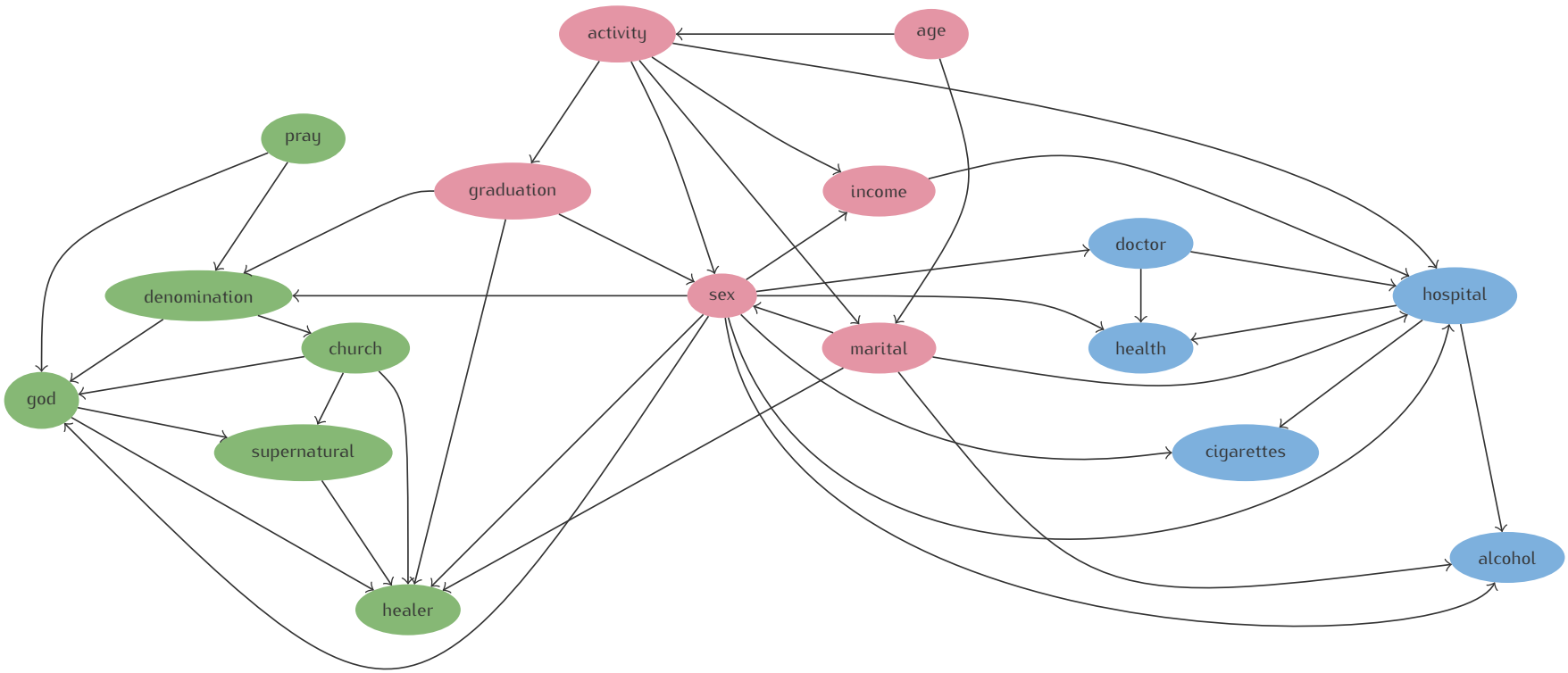
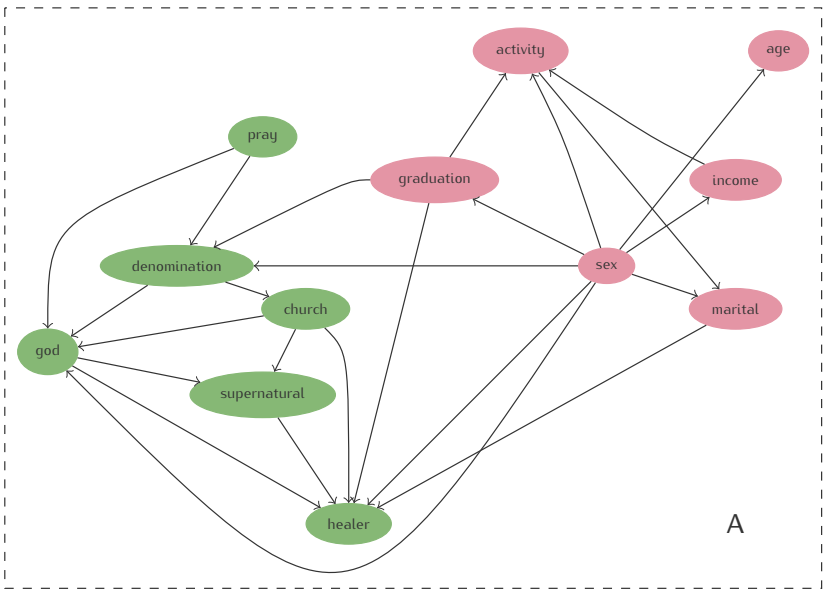
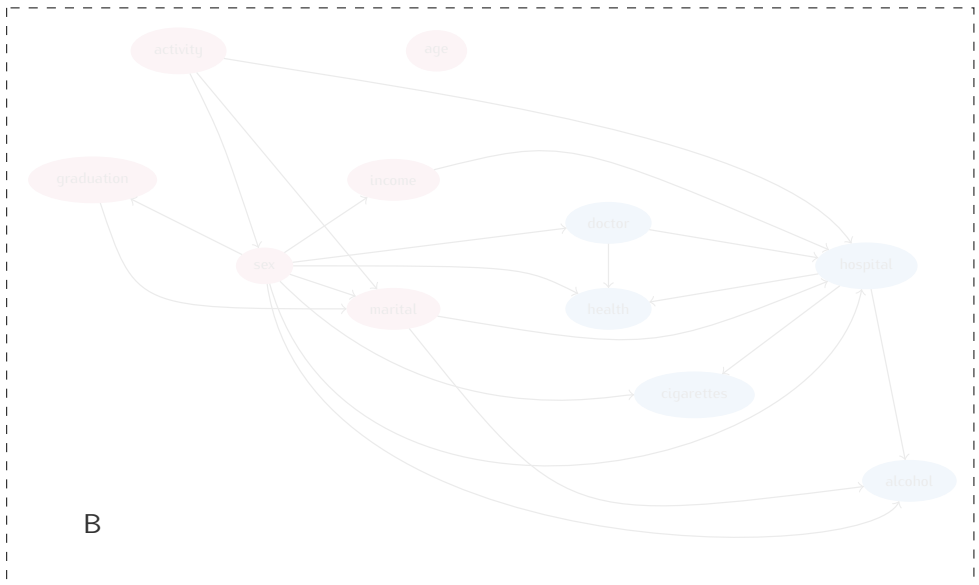


Step 1: estimation and combination of the graph structures

individual graph structures for the common variables





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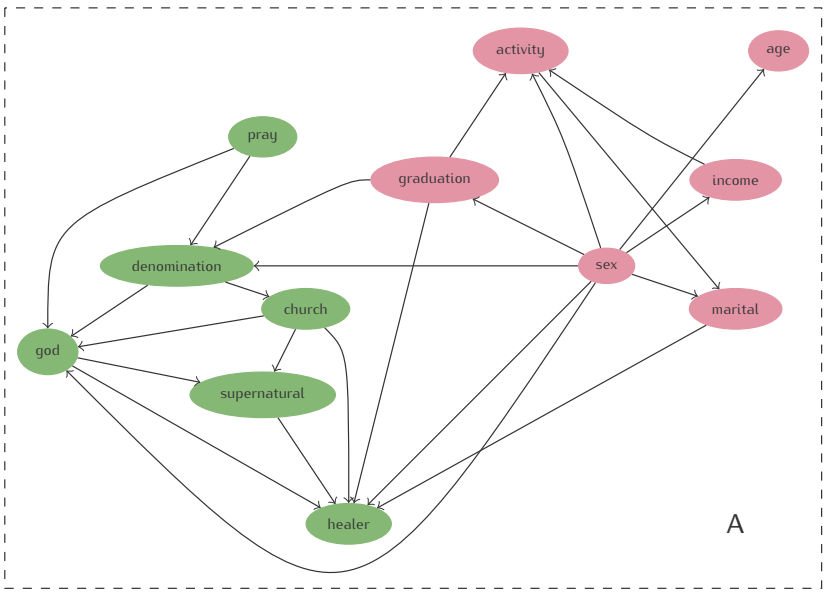
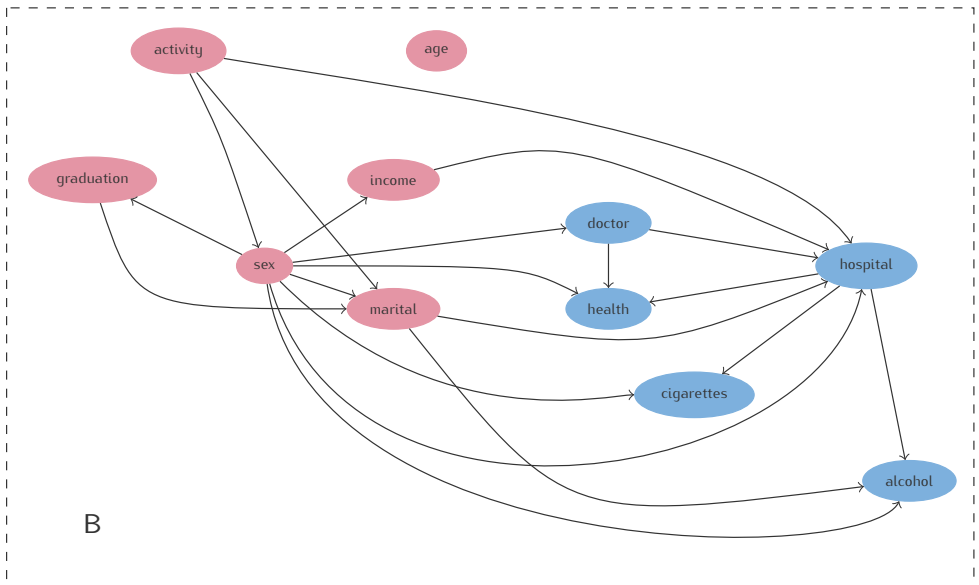
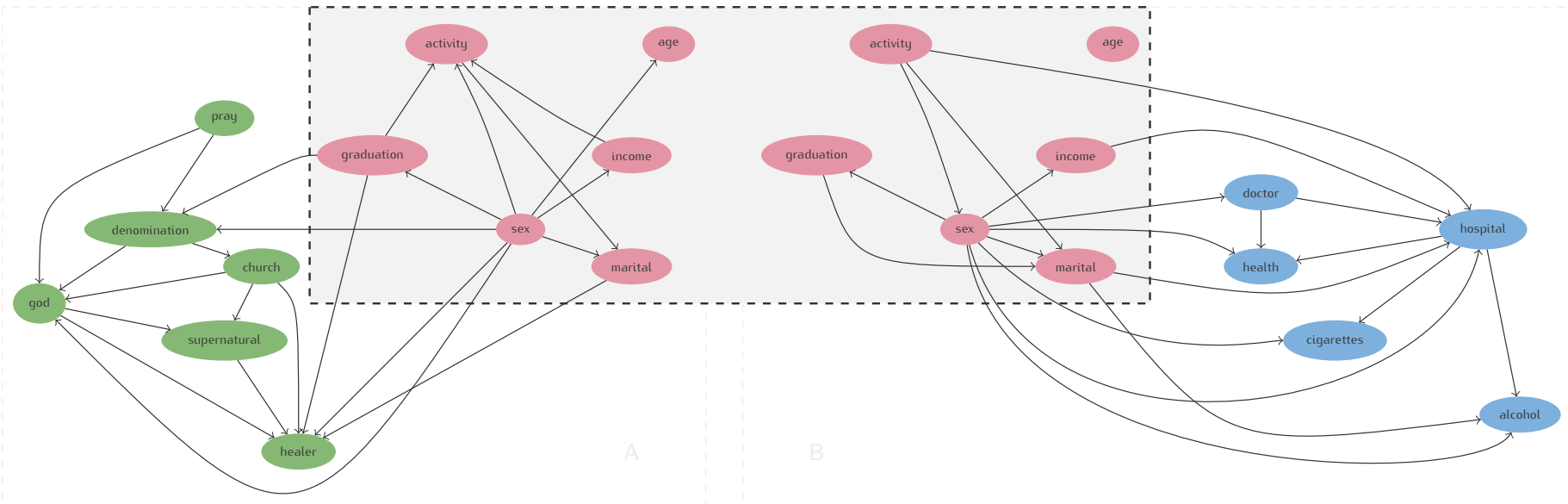




Step 1: estimation and combination of the graph structures

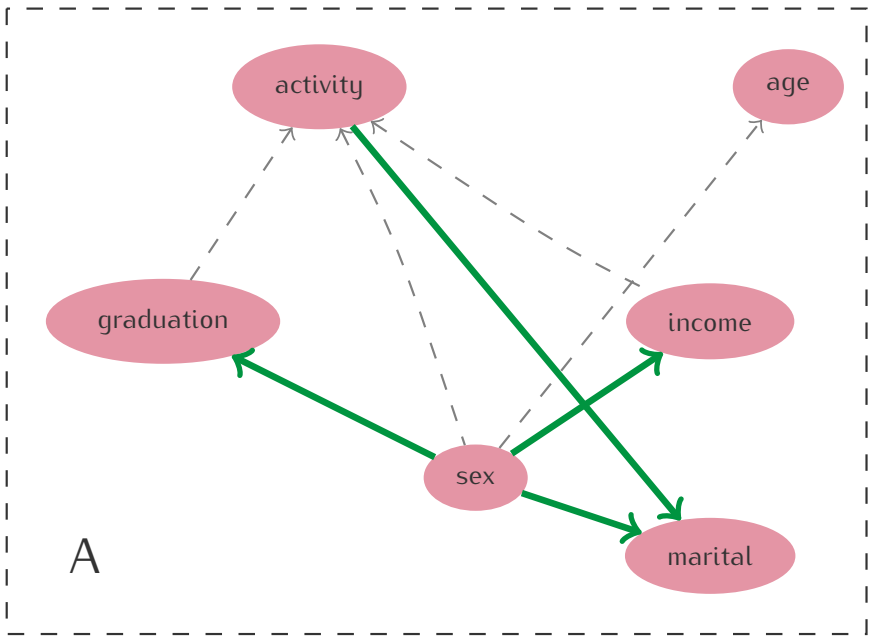
individual graph structures for the common variables

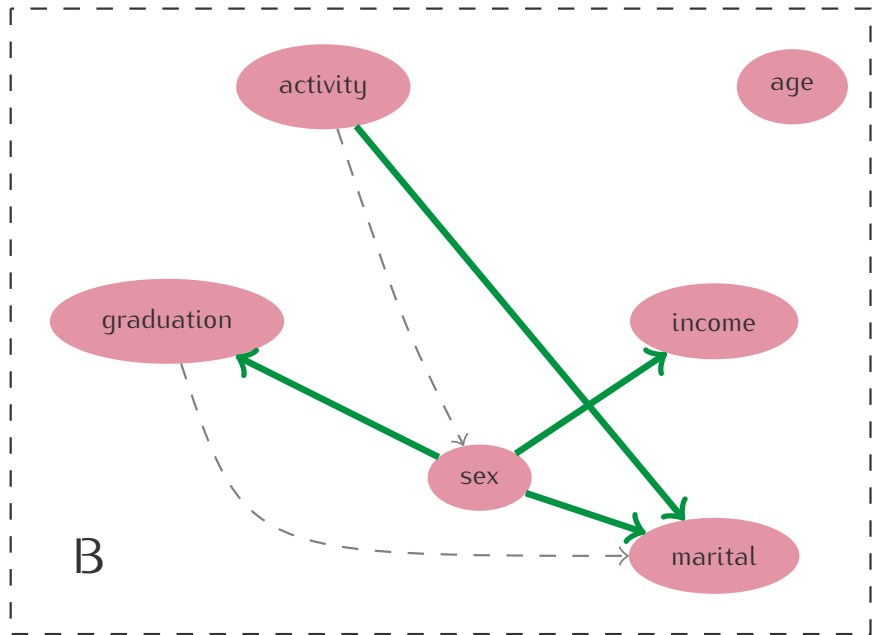


Step 1: estimation and combination of the graph structures

individual graph structures for the common variables

edge intersection

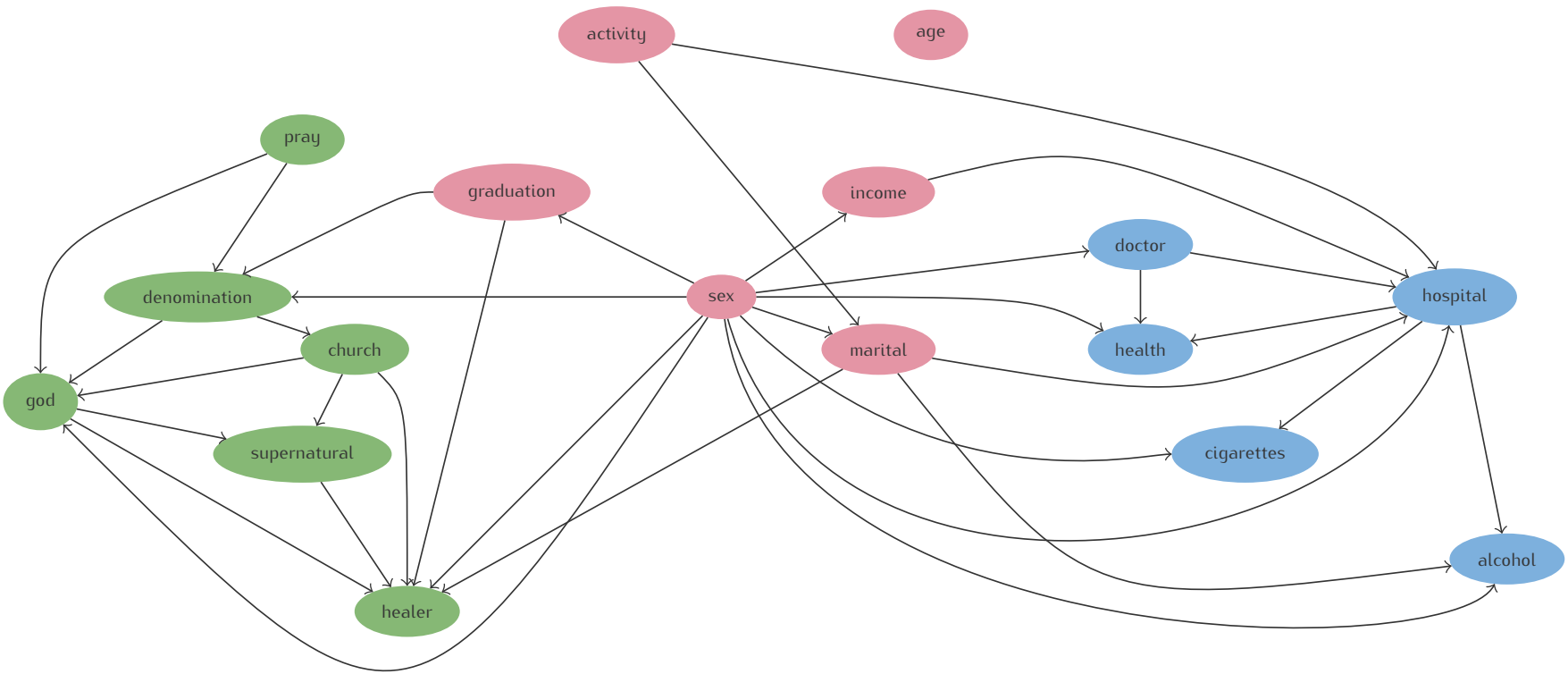




Step 1: estimation and combination of the graph structures

individual graph structures for the common variables

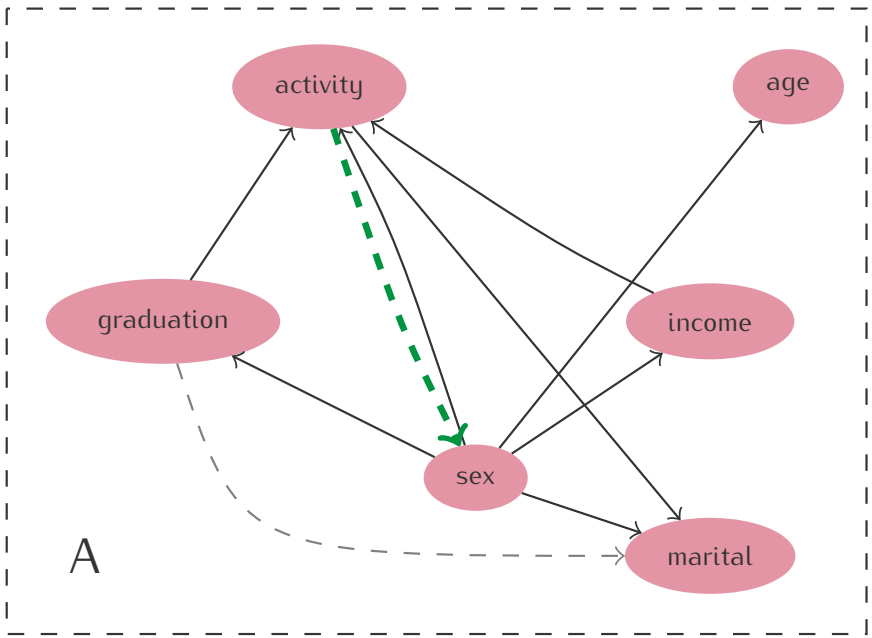
edge intersection

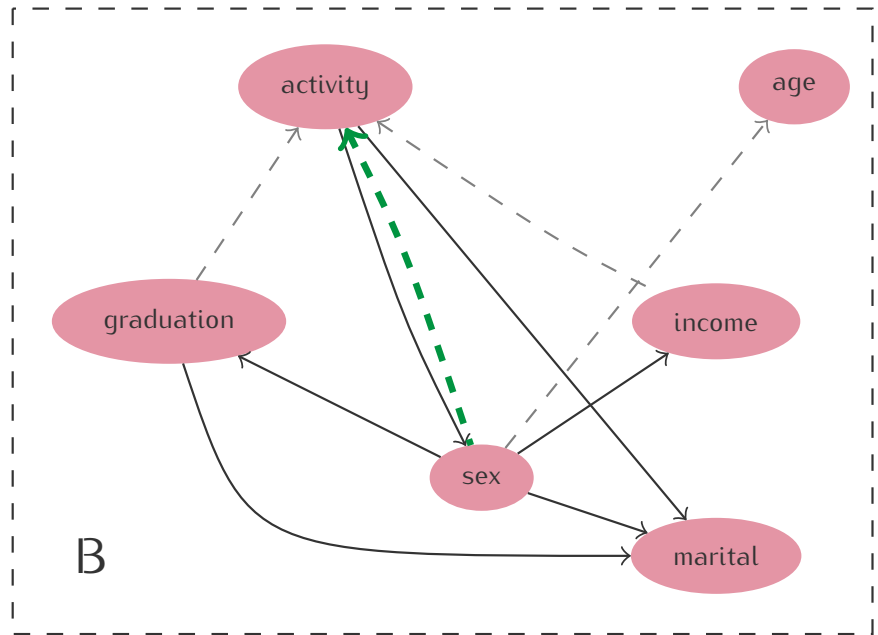


Step 1: estimation and combination of the graph structures

individual graph structures for the common variables

edge union





Step 1: estimation and combination of the graph structures

individual graph structures for the common variables

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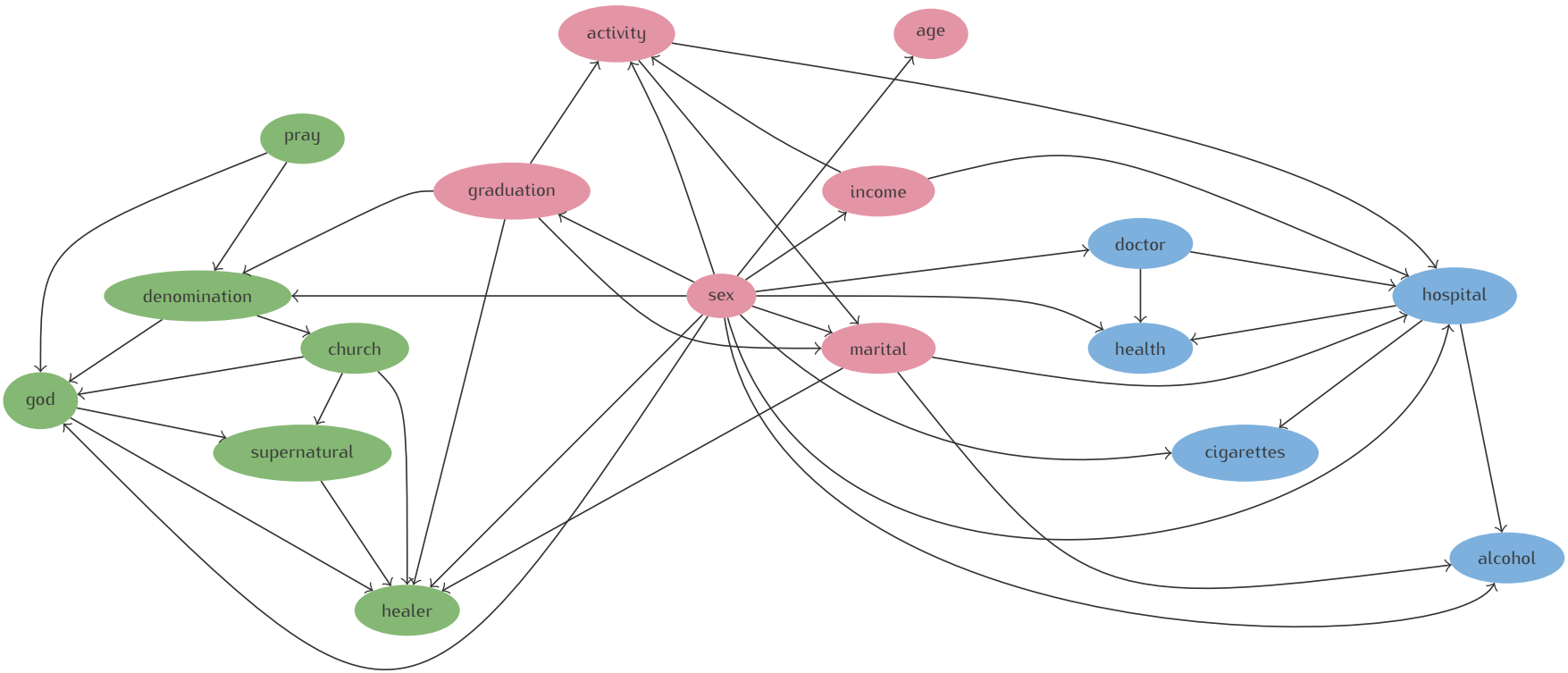

Step 2: estimation of the local parameters and the joint probability distribution

$$\hat{P}^{\mathrm{A}\cup\mathrm{B}}(\mathbf{X}=\mathbf{x},\mathbf{Y}=\mathbf{y},\mathbf{Z}=\mathbf{z}) = \prod_{k=1}^{q} \hat{p}_{\hat{\mathcal{G}}^{\mathrm{A}}_{\mathbf{X},\mathbf{Y}}}(y_k|\mathbf{pa}(Y_k)) \cdot \prod_{\ell=1}^{r} \hat{p}_{\hat{\mathcal{G}}^{\mathrm{B}}_{\mathbf{X},\mathbf{Z}}}(z_\ell|\mathbf{pa}(Z_\ell)) \cdot \prod_{j=1}^{p} \hat{p}_{\hat{\mathcal{G}}^{\mathrm{A}\cup\mathrm{B}}_{\mathbf{X}}}(x_j|\mathbf{pa}(X_j))$$

Step 3: imputation of the missing values

- data set A: draw synthetic values for Z_ℓ, $\ell = 1,\ldots,r$, given the realizations of $\mathbf{X}$ for every $i \in \mathcal{I}_A$ from the estimated posterior distribution $\hat{P}^{\mathrm{A}\cup\mathrm{B}}(\mathbf{Z}=\mathbf{z}|\mathbf{X}=\mathbf{x})$
- data set B: draw synthetic values for Y_k, $k = 1,\ldots,q$ given the realizations of $\mathbf{X}$ for every $i \in \mathcal{I}_B$ from the estimated posterior distribution $\hat{P}^{\mathrm{A}\cup\mathrm{B}}(\mathbf{Y}=\mathbf{y}|\mathbf{X}=\mathbf{x})$

Representativeness of the synthetic file

(Rässler (2002))

- Four quality levels
  1. preserving marginal distributions
  2. preserving correlation/association structures
  3. preserving the joint distribution
  4. preserving the individual values

Jenson-Shannon divergence

denomination	church	god	supernatural	healer	pray
0.000	0.001	0.001	0.004	0.021	0.002

doctor	hospital	cigarettes	alcohol	health
0.001	0.003	0.004	0.001	0.000

Representativeness of the synthetic file

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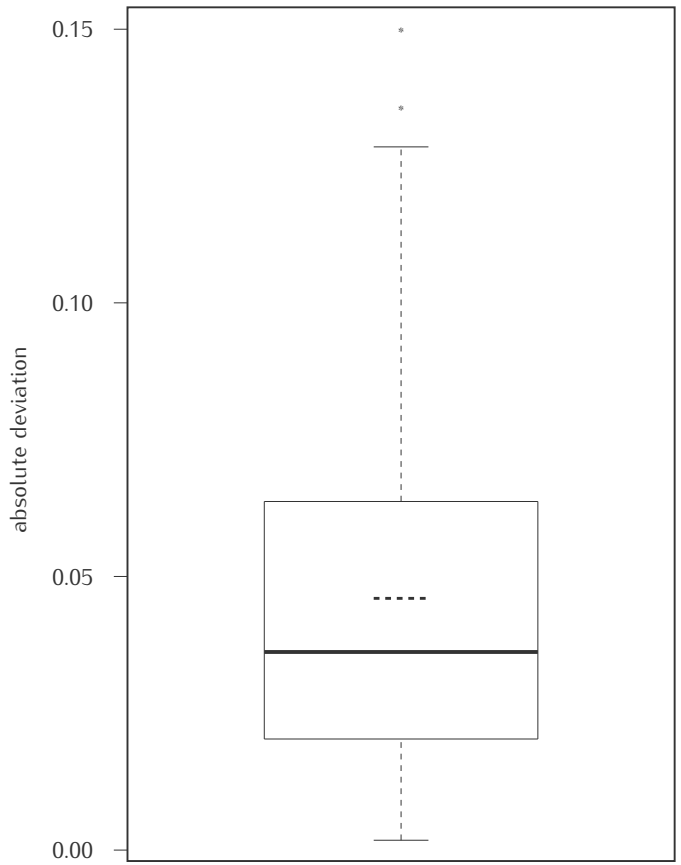

corrected contingency coefficient

Conclusion and outlook

- Representation of conditional independence assumption in the framework of statistical matching by probabilistic graphical models
- Incorporation of further information about the conditional independence structure achieved by the Bayesian network approach
- Marginal distributions and association structure among variables are well preserved

- Simulation studies
- Extension for continuous and mixed discrete/continuous data
- Undirected probabilistic graphical models
- Inclusion of auxiliary information
- Imprecise probabilistic graphical models

References (selection)

D'Orazio, M., Di Zio, M. and Scanu, M. (2006). *Statistical Matching: Theory and Practice*, Wiley, Chichester, United Kingdom.

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