QBism, or taking Wigner's friend seriously

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The B in QBism

- Bayesian?

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- Bayesian? **NO**

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- Bruno de Finetti?

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- B? **YES!**

Four tenets of QBism

1. All probabilities, including those equal to zero or one, are valuations that an agent ascribes to his or her degrees of belief in possible outcomes.

2. The Born rule is normative, not descriptive or prescriptive.

3. Quantum measurement outcomes are personal experiences for the agent gambling on them.

4. A measurement apparatus is conceptually an extension of the agent.

Tenet 1

1. All probabilities, including those equal to zero or one, are valuations that an agent ascribes to his or her degrees of belief in possible outcomes.

- Decision-theoretic, personalist probabilities
- No requirement that different agents assign the same probabilities
- Probabilities are not determined by a systems real properties (“λ”)

Tenet 2

2. The Born rule is normative, not descriptive or prescriptive.

- It's not a law that constrains how nature behaves.
- It is a consistency requirement that guides an agent's decision making.

Tenet 3

3. Quantum measurement outcomes are personal experiences for the agent gambling on them.

- Measurement outcomes are not objective facts.
- Wigner's friend

Tenet 4

4. A measurement apparatus is conceptually an extension of the agent.

- Agent $\neq$ Person
- An agent is an entity that is capable of using the Born rule normatively.

Three routes to QBism

Route 1 starts with de Finetti 1931

Route 2 starts with Einstein 1935

Route 3 starts with Wigner 1961

Route 3, starting with Wigner 1961

Wigner's friend makes a measurement

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In QBism the paradox does not arise:

In QBism, the friend's measurement outcome is personal to the friend.

Route 3, starting with Wigner 1961

Wigner 1961: Wigner's Friend

Caves, Fuchs, RS 2007: *Subjective probability and quantum certainty*: "facts for the agent"

Fuchs, Mermin, RS 2014: *An Introduction to QBism with an Application to the Locality of Quantum Mechanics*: "experience"

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no-go theorems by Brukner 2015, Frauchiger and Renner 2016: strengthen the QBist case

Route 3, starting with Wigner 1961

"[...] the new threat is to interpretations that try to have their cake and eat it: an exact Schroedinger equation and objective, single measurement outcomes.

On the other hand, there are at least two interpretations for which the result is actually good news: QBism and relational quantum mechanics. Proponents of both have always insisted that a measurement result is only real for the agent or system that experiences it. What felt, for me at least, as squeamishness about recognizing what seem like unproblematically objective facts, appears now to be crucial to the interpretation's consistency."

(Matthew Pusey in Nature Physics, commenting on Frauchiger and Renner)

Route 2, starting with Einstein

Einstein 1935 (letter to Schrödinger): assuming λ and locality $\Longrightarrow$ ψ is not a function of λ.

Bell 1961: assuming λ and locality and quantum theory $\Longrightarrow$ contradiction

QBism: rejects λ (and hence reject the EPR criterion of reality)

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recent no-go theorems strengthen the case for QBism

Lambda

The assumption of an ontological model:

For any measurement on a physical system, either the outcomes or their probabilities are determined by the system's real properties, λ. (Harrigan and Spekkens, 2007).

(Potentially misleading alternative labels for the same idea: "hidden variables", "realism".)

Einstein to Schrödinger (1935, not EPR)

Consider the state $|\psi^{AB}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)$,

where $|0\rangle$ and $|1\rangle$ are the eigenstates of the spin Z operator.

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Einstein:

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Implication, assuming locality (Caves,Fuchs,RS 2002):

$|\psi^B\rangle$ is not a function of "the real state at B", i.e., $|\psi^B\rangle$ is not a real property of the system at B.

Loophole-free Bell experiment

You have to give up either locality or λ.

QBism rejects λ.

Route 1, starting with de Finetti

de Finetti 1931: *Probabilismo.* "probability does not exist"

Savage 1954: *The Foundations of Statistics*. Probability from decision theory

Caves, Fuchs, RS 2002: Quantum probabilities as Bayesian probabilities

Spekkens 2004: *In defense of the epistemic view of quantum states: a toy theory*, gives compelling arguments for an epistemic view of quantum states, even if from an ontological perspective

Fuchs, RS 2013: *Quantum Bayesian coherence*, personalist probability in QBism fully spelled out

Decision-theoretic personalist probability

My probability of an event E is p if I regard \$$p$ as the fair price of a lottery ticket that pays \$1 if E occurs.

Consistency (or Dutch-book coherence)

My probabilities (i.e., valuations) do not satisfy the rules of probability theory
$\Longrightarrow$ Sure Loss

Example: $p = 1.5$ ("\$1.50 is a fair price for a ticket that pays \$1 in the best case.")

Probability theory as a tool to detect inconsistency

3 coins are tossed.

Example 1: $p(HHH) = 0.5$ OK

Example 2: $p(\text{at least } 2H) + p(\text{at least } 2T) = 0.9$ not OK

- Probability theory provides relations between probabilities.
- It does not tell you what the correct probabilities are.

Normative versus descriptive rules

Example (Allais):

1. Given a choice between 1a or 1b, which do you choose?
2. Given a choice between 2a or 2b, which do you choose?

	1a	**1b**	**2a**	**2b**
payoff	p	p	p	p
0		0.01		
$\$5 \times 10^6$	1	0.89		
$\$10 \times 10^6$		0.10		

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"Normative": How should you act?

"Descriptive": How do real agents act?

Frequencies and repeated trials

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Exchangeability characterises repeated trials

For N trials,

$$\rho^{(N)} \text{ exchangeable} \Longrightarrow \rho^{(N)} = \int \rho \otimes \rho \otimes \ldots \otimes \rho \, d\rho$$

(this is the quantum de Finetti theorem)

Quantum Baysianism?

Bayes rule $p_{\mathrm{Wed}}(AB) = p_{\mathrm{Wed}}(A|B)p_{\mathrm{Wed}}(B)$ follows from consistency (no sure loss)

Assume you observe B on Thursday morning:

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Bayesian updating is not fundamental!

QBism should really be Quantum Bettabilitarianism (following Oliver Wendell Holmes)

The Born as a normative consistency requirement

Example:

Consider a spin-1/2 particle and consider making one of four spin measurements:

(1) along $\vec{x}$; (2) along $\vec{y}$; (3) along $\vec{z}$; (4) along $\frac{1}{\sqrt{3}}(\vec{x}+\vec{y}+\vec{z})$

Assume the corresponding probabilities for "up" are p_1, p_2, p_3, p_4.

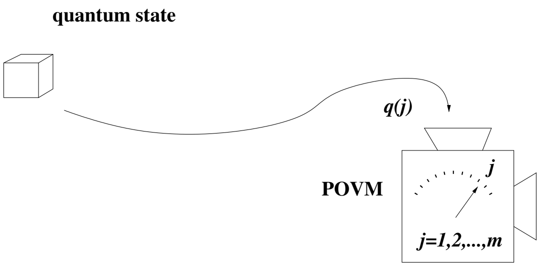
The Born rule constrains these probabilities (any three, if allowed, determine the fourth).

The Born rule provides relations between probabilities. It does not tell you what the correct probabilities are.

The Born as a normative consistency requirement

Born rule

$$q(j) = \mathrm{tr}(\rho E_j)$$



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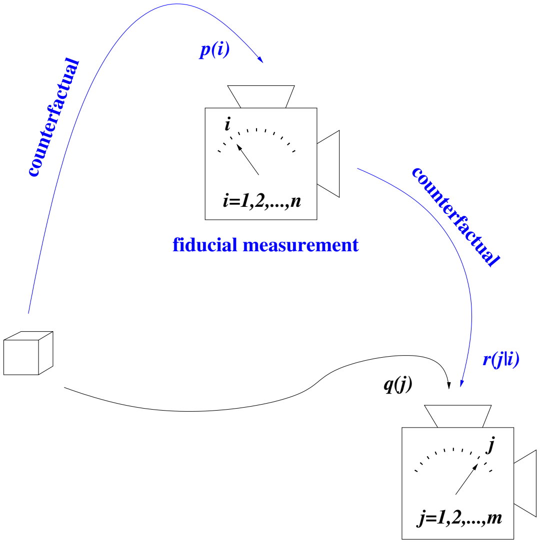

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quantum state

$\rho \longleftrightarrow p(i)$

POVM

$\{E_j\} \longleftrightarrow r(j|i)$

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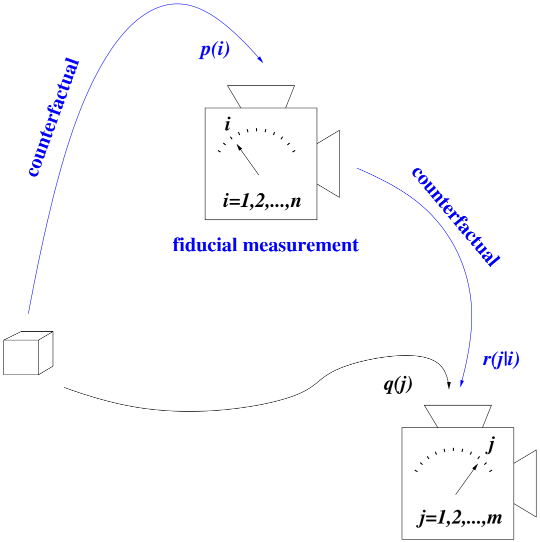

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Born rule, rewritten

$q(j) = f(p(i), r(j|i))$

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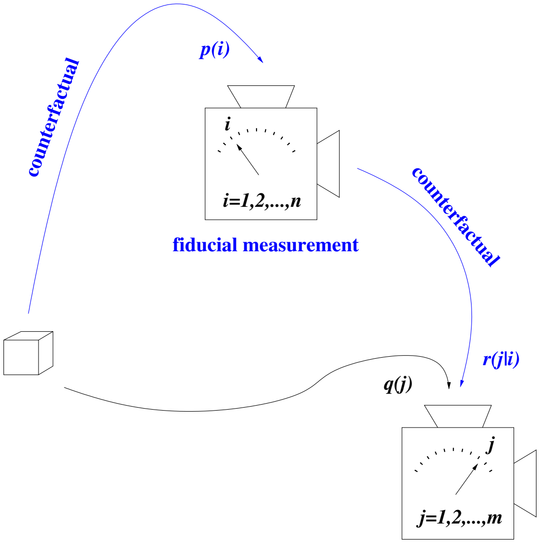

Born rule, rewritten

$$q(j) = f\big(p(i), r(j|i)\big)$$

Back to Wigner's friend: Frauchiger and Renner

Four players: Wigner and his assistant make measurements on two labs each containing a friend.

Players make seemingly unassailable statements about other players' measurement results and their predictions.

A contradiction results.

Frauchiger and Renner's Assumption (C)

A theory T that satisfies Assumption (C) allows any agent A to reason as follows. If A has established $s_1^A =$ "I am certain at time t_0 that agent B, upon reasoning using T, is certain that $x = \xi$ at time t." for some x, ξ, and $t > t_0$, then A can conclude $s_2^A =$ "I am certain at time t_0 that $x = \xi$ at time t."

(version from April 2018 preprint)

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$s_1^A =$ "I am certain that λ"

$\lambda =$ "agent B is certain that $x = \xi$"

Assumption (C) in (almost) QBist terms

Let A and B be two agents who start with the same prior knowledge about the experiment. Assumption (C) allows A to transform any statement of the form

"I am certain that, if I checked B's notebook at some time t_0, I would read that he or she is certain that statement s holds."

into

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Assumption (C), viewed as a general rule, is incompatible with QBism.

Can a QBist take other agents' views into account?

Yes.

But assumption (C) should be regarded as a contingent assumption applying to a specific agent in a specific situation.

It should be abandoned if it clashes with the Born rule.