# Bayesian inference under ambiguity: Conditional prior belief functions

G. Coletti[a] and D. Petturiti[a] and B. Vantaggi[b]

[a] University of Perugia
[b] "Sapienza" University of Roma

**ISIPTA 2017**
**Lugano, 10-14 July 2015**

# AIM

To study Bayesian inference under imprecise prior information: the starting point is a precise strategy $\sigma$ and a full B-conditional prior belief function $Bel_B$, conveying ambiguity in probabilistic prior information.

The prior knowledge could be only partially specified or, even worse, it could refer to a different space of hypotheses.

Instead of considering a single prior distribution, one is forced to take into account a set of priors (see, e.g., Dempster 1967, DeRoberts-Hartigan 1981, Huber 1981, Gilboa Schmeidler 1989, Wasserman 1990, Wasserman-Kadane 1990, Walley 1991, Chateauneuf et al. 2001, Klibanoff-Hanany 2007).

# Applications of multi-priors

- Statistics: Partial identifiable models, Models with latent variables (mixture models), Hierarchical Bayesian moles, Nuisance parameters elimination, Models with misclassified variables, Elicitation of priors
- Economic theory: Gilboa-Schmeidler decision model, Ambiguity in decision theory and in game theory
- Probability: de Finetti coherent probabilities, Random sets, Multivalued-mappings, Imprecise probabilities,

# Non-additive uncertainty measures

$\varphi : \mathcal{A} \to [0,1]$ s.t. $\varphi(\emptyset) = 0$, $\varphi(\Omega) = 1$ uncertainty measure:

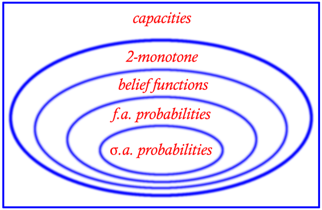
capacity: $A \subseteq B \Rightarrow \varphi(A) \leq \varphi(B)$;

$n$-monotone: $\varphi\left(\bigvee_{i=1}^{n} E_i\right) \geq \sum_{\emptyset \neq I \subseteq \{1,\ldots,n\}} (-1)^{|I|+1} \varphi\left(\bigwedge_{i \in I} E_i\right)$;

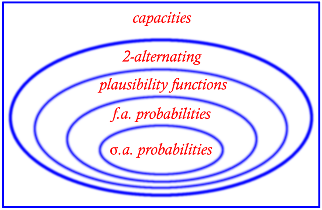
belief function: $n$-monotone for $n \in \mathbb{N}$, $n \geq 2$.

$\overline{\varphi} : \mathcal{A} \to [0,1]$, $\overline{\varphi}(A) = 1 - \varphi(A^c)$ for every $A \in \mathcal{A}$, dual measure.

$\varphi : \mathcal{A} \to [0,1]$




$\overline{\varphi} : \mathcal{A} \to [0,1]$



# Belief function: conditioning

Conditioning for belief function is deeply discussed in literature (Dempster AMS 1967 JRSS 1968), (see also Dubois-Denœux 2012, Fagin-Halpern 1991, Jaffray IEEE 1992) have been introduced through a *generalized Bayesian conditioning rule* discussed also in (Walley TR 1981) for 2-monotone capacities.

If $Bel(E \wedge H) + Pl(E^c \wedge H) > 0$

$$Bel_B(E|H) = \frac{Bel(E \wedge H)}{Bel(E \wedge H) + Pl(E^c \wedge H)},$$

# Conditional belief[2]

**Definition**

A function $Bel_B : \mathcal{A} \times \mathcal{A}^0 \to [0,1]$[1] is a **full B-conditional belief function** on $\mathcal{A}$ if there exists a C-class $\{Bel_0, \dots, Bel_k\}$ of belief functions on $\mathcal{A}$ such that, for every $E|H \in \mathcal{A} \times \mathcal{A}^0$, if $E \wedge H = H$ then $Bel_B(E|H) = 1$, while if $E \wedge H \neq H$

$$Bel_B(E|H) = \frac{Bel_{\alpha_{E,H}}(E \wedge H)}{Bel_{\alpha_{E,H}}(E \wedge H) + Pl_{\alpha_{E,H}}(E^c \wedge H)}, \quad (1)$$

where $\{Pl_0, \dots, Pl_k\}$ is the set of dual plausibility functions of $\{Bel_0, \dots, Bel_k\}$ and

$$\alpha_{E,H} = \min\{\alpha \in \{0, \dots, k\} : Bel_\alpha(E \wedge H) + Pl_\alpha(E^c \wedge H) > 0\}$$

---

[1] $\mathcal{A}^0 = \mathcal{A} \setminus \{\emptyset\}$

[2] Coletti et al. Inf. Science 2016

# Full B-conditional belief

These conditional measures $Bel_B$ and $Pl_B$ determine the non-empty compact set

$\mathcal{P}_B = \{\tilde{\pi} \ : \ \tilde{\pi}$ is a full conditional probability on $\mathcal{A}$, $Bel_B \leq \tilde{\pi} \leq Pl_B\}$,

$$Bel_B = \min \mathcal{P}_B \quad Pl_B = \max \mathcal{P}_B$$

For every $Bel_B : \mathcal{A} \times \mathcal{A}^0 \to [0,1]$ there is a finite Boolean algebra $\mathcal{B}$ and a full conditional probability $P : \mathcal{B} \times \mathcal{B}^0 \to [0,1]$ such that $\mathcal{P}_B$ can be recovered as the set of coherent extensions of $P$ to $\mathcal{A} \times \mathcal{A}^0$ and, thus,

$$Bel_B = \min \mathcal{P}_B \qquad Pl_B \max \mathcal{P}_B$$

# Full B-conditional belief

These conditional measures $Bel_B$ and $Pl_B$ determine the non-empty compact set

$\mathcal{P}_B = \{\tilde{\pi} \ : \ \tilde{\pi}$ is a full conditional probability on $\mathcal{A}$, $Bel_B \leq \tilde{\pi} \leq Pl_B\}$,

$$Bel_B = \min \mathcal{P}_B \quad Pl_B = \max \mathcal{P}_B$$

For every $Bel_B : \mathcal{A} \times \mathcal{A}^0 \to [0,1]$ there is a finite Boolean algebra $\mathcal{B}$ and a full conditional probability $P : \mathcal{B} \times \mathcal{B}^0 \to [0,1]$ such that $\mathcal{P}_B$ can be recovered as the set of coherent extensions of $P$ to $\mathcal{A} \times \mathcal{A}^0$ and, thus,

$$Bel_B = \min \mathcal{P}_B \qquad Pl_B \max \mathcal{P}_B$$

# Bayesian statistics

In the classical Bayesian setting[3]

- $\pi : \mathcal{A}_{\mathcal{L}} \to [0,1]$, (finitely additive) prior probability;
- $\sigma : \mathcal{A} \times \mathcal{L} \to [0,1]$, strategy s.t. for every $H_i \in \mathcal{L}$

  **(S1)** $\sigma(F|H_i) = 1$ if $F \wedge H = H$ for $F \in \mathcal{A}$;

  **(S2)** $\sigma(\cdot|H_i)$ is a finitely additive probability on $\mathcal{A}$;
- $\lambda = \sigma_{|\mathcal{A}_{\mathcal{E}} \times \mathcal{L}}$, statistical model

$\Rightarrow$ $\{\pi, \lambda\}$ and $\{\pi, \sigma\}$ is a coherent conditional probability

[3] $\mathcal{L} = \{H_i\}_{i \in I}, \mathcal{E} = \{E_j\}_{j \in J}$, partitions; $\mathcal{A}_{\mathcal{L}}, \mathcal{A}_{\mathcal{E}}$, Boolean algebras with $\langle \mathcal{L} \rangle \subseteq \mathcal{A}_{\mathcal{L}} \subseteq \langle \mathcal{L} \rangle^*$, $\langle \mathcal{E} \rangle \subseteq \mathcal{A}_{\mathcal{E}} \subseteq \langle \mathcal{E} \rangle^*$

# The role of coherence in Bayesian statistics

Given a statistical model $\lambda$ on $\mathcal{A}_{\mathcal{E}} \times \mathcal{L}$ and $\mathcal{A} = \langle \mathcal{A}_{\mathcal{L}} \cup \mathcal{A}_{\mathcal{E}} \rangle$, then there exists a unique strategy $\sigma$ on $\mathcal{A} \times \mathcal{L}$ such that $\sigma_{|\mathcal{A}_{\mathcal{E}} \times \mathcal{L}} = \lambda$.

An aim is to determine the lower and upper envelope of the coherent extensions $\tilde{P}$ of $\{\sigma, \pi\}$[4].

[4] Petturiti-V. IJAR 2017

# Bayes theorem under ambiguity

- $Bel_B$ is a full B-conditional belief function on $\mathcal{A}_{\mathcal{L}}$[5];
- $\sigma : \mathcal{A} \times \mathcal{L} \to [0,1]$, strategy s.t. for every $H_i \in \mathcal{L}$

  **(S1)** $\sigma(F|H_i) = 1$ if $F \wedge H = H$ for $F \in \mathcal{A}$;

  **(S2)** $\sigma(\cdot|H_i)$ is a finitely additive probability on $\mathcal{A}$;
- $\lambda = \sigma_{|\mathcal{A}_{\mathcal{E}} \times \mathcal{L}}$, statistical model

$\Rightarrow$ $\sigma$ is a strategy on $\mathcal{A} \times \mathcal{L}$

$\mathcal{P}_B = \{\tilde{\pi} \ : \ \tilde{\pi}$ is a full conditional probability on $\mathcal{A}$, $Bel_B \leq \tilde{\pi} \leq Pl_B\}$,

[5] Coletti et. al Inf. Science 2016

# Bayes theorem under ambiguity

- $Bel_B$ is a full B-conditional belief function on $\mathcal{A}_{\mathcal{L}}$[5];
- $\sigma : \mathcal{A} \times \mathcal{L} \to [0,1]$, strategy s.t. for every $H_i \in \mathcal{L}$
  **(S1)** $\sigma(F|H_i) = 1$ if $F \wedge H = H$ for $F \in \mathcal{A}$;
  **(S2)** $\sigma(\cdot|H_i)$ is a finitely additive probability on $\mathcal{A}$;
- $\lambda = \sigma_{|\mathcal{A}_{\mathcal{E}} \times \mathcal{L}}$, statistical model

$\Rightarrow$ $\sigma$ is a strategy on $\mathcal{A} \times \mathcal{L}$

$\mathcal{P}_B = \{\tilde{\pi} \ : \ \tilde{\pi}$ is a full conditional probability on $\mathcal{A}$, $Bel_B \leq \tilde{\pi} \leq Pl_B\}$,

[5] Coletti et. al Inf. Science 2016

# Bayes theorem under ambiguity

⇒ $Bel_B$ is a full B-conditional belief function on $\mathcal{A}_\mathcal{L}$
⇒ $\mathcal{P}_B$ is the set of full conditional probabilities on $\mathcal{A}_\mathcal{L}$ dominating $Bel_B$
⇒ $\sigma$ is a strategy on $\mathcal{A} \times \mathcal{L}$

$\mathcal{P} = \{\tilde{P} \ : \ \tilde{P}$ is a full cond. prob. on $\mathcal{A}$ extending $\{\tilde{\pi}, \sigma\}$, $\tilde{\pi} \in \mathcal{P}_B\}$,

is a non-empty compact subset of $[0,1]^{\mathcal{A} \times \mathcal{A}^0}$ endowed with the product topology and

$$\underline{P} = \min \mathcal{P} \quad \overline{P} = \max \mathcal{P}$$

⇒ The lower envelope $\underline{P}(\cdot|\cdot)$ turns out to be the natural extension of the Williams-coherent lower conditional probability $\{Bel_B, \sigma\}$.
⇒ In the finite setting it coincides with that due to (Walley 1991) since the conglomerability condition is automatically satisfied.

# Bayes theorem under ambiguity

⇒ $Bel_B$ is a full B-conditional belief function on $\mathcal{A}_{\mathcal{L}}$
⇒ $\mathcal{P}_B$ is the set of full conditional probabilities on $\mathcal{A}_{\mathcal{L}}$ dominating $Bel_B$
⇒ $\sigma$ is a strategy on $\mathcal{A} \times \mathcal{L}$

$\mathcal{P} = \{\tilde{P} \;:\; \tilde{P}$ is a full cond. prob. on $\mathcal{A}$ extending $\{\tilde{\pi}, \sigma\}$, $\tilde{\pi} \in \mathcal{P}_B\}$,

is a non-empty compact subset of $[0,1]^{\mathcal{A} \times \mathcal{A}^0}$ endowed with the product topology and

$$\underline{P} = \min \mathcal{P} \quad \overline{P} = \max \mathcal{P}$$

⇒ The lower envelope $\underline{P}(\cdot|\cdot)$ turns out to be the natural extension of the Williams-coherent lower conditional probability $\{Bel_B, \sigma\}$.
⇒ In the finite setting it coincides with that due to (Walley 1991) since the conglomerability condition is automatically satisfied.

# Bayes theorem under ambiguity

⇒ $Bel_B$ is a full B-conditional belief function on $\mathcal{A}_{\mathcal{L}}$
⇒ $\mathcal{P}_B$ is the set of full conditional probabilities on $\mathcal{A}_{\mathcal{L}}$ dominating $Bel_B$
⇒ $\sigma$ is a strategy on $\mathcal{A} \times \mathcal{L}$

$\mathcal{P} = \{\tilde{P} \ : \ \tilde{P}$ is a full cond. prob. on $\mathcal{A}$ extending $\{\tilde{\pi}, \sigma\}$, $\tilde{\pi} \in \mathcal{P}_B\}$,

is a non-empty compact subset of $[0,1]^{\mathcal{A} \times \mathcal{A}^0}$ endowed with the product topology and

$$\underline{P} = \min \mathcal{P} \quad \overline{P} = \max \mathcal{P}$$

⇒ The lower envelope $\underline{P}(\cdot|\cdot)$ turns out to be the natural extension of the Williams-coherent lower conditional probability $\{Bel_B, \sigma\}$.
⇒ In the finite setting it coincides with that due to (Walley 1991) since the conglomerability condition is automatically satisfied.

# Bayes Theorem under ambiguity

The lower envelope $\underline{P}(\cdot|\cdot)$ is such that, for every $F|K \in \mathcal{A} \times \mathcal{A}^0_{\mathcal{L}}$, if $F \wedge K = K$, then $\underline{P}(F|K) = 1$, otherwise:

*(i)* if $K \in \mathcal{A}^0_{\mathcal{L}}$, then

$$\underline{P}(F|K) = \oint \sigma(F|H_i) Bel_B(\mathrm{d}H_i|K);$$

*(ii)* if $K \in \mathcal{A}^0 \setminus \mathcal{A}^0_{\mathcal{L}}$, then if there exists $A \in \mathcal{A}^0_{\mathcal{L}}$ such that $K \subseteq A$ and $\underline{P}(K|A) > 0$ we have that

$$\underline{P}(F|K) = \min\left\{ \frac{\underline{P}(F \wedge K|A)}{\underline{P}(F \wedge K|A) + U(F^c, K; A)}, \frac{L(F, K; A)}{L(F, K; A) + \overline{P}(F^c \wedge K|A)} \right\},$$

otherwise $\underline{P}(F|K) = 0$.

where

$$L(F, K; A) = \min_{\tilde{\pi} \in \mathcal{P}_B} \left\{ \sum_{i=1}^n \sigma(FK|H_i)\tilde{\pi}(H_i|A) : \sum_{i=1}^n \sigma(F^c K|H_i)\tilde{\pi}(H_i|A) = \overline{P}(F^c K|A) \right\},$$

$$U(F, K; A) = \max_{\tilde{\pi} \in \mathcal{P}_B} \left\{ \sum_{i=1}^n \sigma(FK|H_i)\tilde{\pi}(H_i|A) : \sum_{i=1}^n \sigma(F^c K|H_i)\tilde{\pi}(H_i|A) = \underline{P}(F^c K|A) \right\},$$

# Bayes Theorem under ambiguity

## Lower posterior probabilities

For every $F|K \in \mathcal{A} \times \mathcal{A}^0$ such that $F \wedge K \neq K$, $K \in \mathcal{A}^0 \setminus \mathcal{A}^0_{\mathcal{L}}$ and there exists $A \in \mathcal{A}^0_{\mathcal{L}}$ such that $K \subseteq A$ and $\underline{P}(K|A) > 0$, if $X(\cdot) = \sigma(F \wedge H|\cdot)$ and $(1 - Y(\cdot)) = (1 - \sigma(F^c \wedge H|\cdot))$ are comonotonic[6] then

$$\underline{P}(F|K) = \frac{\underline{P}(F \wedge K|A)}{\underline{P}(F \wedge K|A) + \overline{P}(F^c \wedge K|A)}.$$

⇒ This is a generalization of a result of (Wasserman 1990)

⇒ $Bel_B(\cdot|K)$ is a belief function on $\mathcal{A}_{\mathcal{L}}$, for every $K \in \mathcal{A}^0_{\mathcal{L}}$

⇒ The function $\underline{P}(\cdot|K)$ can fail 2-monotonicity, for some $K \in \mathcal{A}^0$.

---

[6] $X(\cdot) = \sigma(F \wedge H|\cdot)$ and $(1 - Y(\cdot)) = (1 - \sigma(F^c \wedge H|\cdot))$ defined on $\mathcal{L}$ are *comonotonic* if, for every $H_h, H_k \in \mathcal{L}$,
$[X(H_h) - X(H_k)] \cdot [(1 - Y(H_h)) - (1 - Y(H_k))] \geq 0$

# Example

An automatic system **S** can assume the states $s_1$, $s_2$, $s_3$ with $\pi^{(0)} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and its evolution is determined by the Markov chain

$$A = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

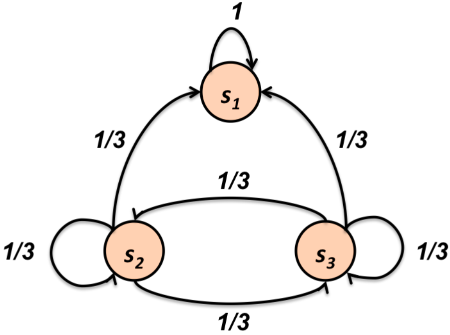



Figure : Transition matrix and graph of the Markov chain related to **S**

# Example

After $n$ steps

$$\pi^{(n)} = \pi^{(n-1)}A = \left(1 - \left(\frac{2}{3}\right)^{n+1}, \frac{1}{3}\left(\frac{2}{3}\right)^{n}, \frac{1}{3}\left(\frac{2}{3}\right)^{n}\right)$$

$\pi^{(n)}$ is positive for every $n \geq 0$, so it induces a unique full cond. probability
The sequence of full cond. probabilities converges pointwise to

| $\mathcal{A}_{\S}$ | $\emptyset$ | $S_1$ | $S_2$ | $S_3$ | $S_1 \vee S_2$ | $S_1 \vee S_3$ | $S_2 \vee S_3$ | $\Omega$ |
|---|---|---|---|---|---|---|---|---|
| $\pi^{(\infty)}(\cdot\|S_1)$ | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| $\pi^{(\infty)}(\cdot\|S_2)$ | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| $\pi^{(\infty)}(\cdot\|S_3)$ | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| $\pi^{(\infty)}(\cdot\|S_1 \vee S_2)$ | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| $\pi^{(\infty)}(\cdot\|S_1 \vee S_3)$ | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| $\pi^{(\infty)}(\cdot\|S_2 \vee S_3)$ | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 1 | 1 |
| $\pi^{(\infty)}(\cdot\|\Omega)$ | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |

that is determined by $\{P_0, P_1\}$ such that $P_0(\cdot) = \pi^{(\infty)}(\cdot|\Omega)$ and $P_1(\cdot) = \pi^{(\infty)}(\cdot|S_2 \vee S_3)$.

# Example

Consider a second system **T** not directly observable: **T** can assume three possible states $t_1$, $t_2$, $t_3$, and if **S** is in state $s_i$ then **T** is not in state $t_i$, for $i = 1, 2, 3$.

$$(B)_* = \bigvee\{S_i \in \S \ : \ S_i \subseteq B\}, \quad Bel_0(B) = P_0((B)_*) \quad \text{and} \quad Bel_1(B) = P_1((B)_*),$$

we obtain a B-conditional belief function on $\mathcal{A}_\Theta$

| $\mathcal{A}_\Theta$ | $\emptyset$ | $T_1$ | $T_2$ | $T_3$ | $T_1 \vee T_2$ | $T_1 \vee T_3$ | $T_2 \vee T_3$ | $\Omega$ |
|---|---|---|---|---|---|---|---|---|
| $Bel_B(\cdot\|T_1)$ | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| $Bel_B(\cdot\|T_2)$ | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| $Bel_B(\cdot\|T_3)$ | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| $Bel_B(\cdot\|T_1 \vee T_2)$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| $Bel_B(\cdot\|T_1 \vee T_3)$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| $Bel_B(\cdot\|T_2 \vee T_3)$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| $Bel_B(\cdot\|\Omega)$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |

# Example

The state of the unobservable system **T** can be verified through a detector **D** taking three possible values $d_1$, $d_2$ and $d_3$, with $d_i$ corresponding to the state $t_i$, for $i = 1, 2, 3$, with a reliability of 90% and equal chances on failures.
The statistical model on $\mathcal{A}_{\mathcal{D}} \times \Theta$

$$\lambda(D_i | T_i) = 90\%, \quad \lambda(D_j | T_i) = \lambda(D_k | T_i) = 5\%$$

$$\underline{P}(T_1 | D_j) = \frac{\underline{P}(T_1 \wedge D_j)}{\underline{P}(T_1 \wedge D_j) + \overline{P}(T_1^c \wedge D_j)} = 0,$$

and $\underline{P}(T_1^c | D_j) = 1$, so, $\underline{P}(T_1 | D_j) = \overline{P}(T_1 | D_j) = 0$ i.e., the observation of the detector **D** does not change our degree of belief on $T_1$

# Example: Nuisance parameter elimination[7]

**PROBLEM: Given a statistical model $\lambda(E|\Theta = \theta, \Gamma = \gamma)$ where $\Theta$ is the interest parameter, we want to eliminate the nuisance parameter $\Gamma$.**

- **Integrated likelihood: for a conditional prior $\pi$**

$$\lambda(E|\Theta = \theta) = \int_{D} \lambda(E|\Theta = \theta, \Gamma = \gamma)\pi(\mathrm{d}(\Gamma = \gamma)|\Theta = \theta)$$

- **Profile likelihood:**

$$\hat{\lambda}(E|\Theta = \theta) = \sup_{\gamma} \lambda(E|\Theta = \theta, \Gamma = \gamma)$$

[7] Berger et al. Stat. Science 1999

# Example: Nuisance parameter elimination (1)

Consider:

- $(\Theta, \Gamma)$, random vector ranging in $\boldsymbol{\Theta} \times \boldsymbol{\Gamma} = \mathbb{N} \times (0, 1)$
- $X = (X_1, \dots, X_k)$, random vector ranging in $\mathbf{X} = \mathbb{N}_0^k$
- $X_i | (\Theta = \theta, \Gamma = \gamma) \sim \mathrm{Bin}(\theta, \gamma)$, for $i = 1, \dots, k$, and independent conditionally to $(\Theta = \theta, \Gamma = \gamma)$
- $\mathcal{L} = \{H_{(\theta,\gamma)} = (\Theta = \theta, \Gamma = \gamma) : (\theta, \gamma) \in \boldsymbol{\Theta} \times \boldsymbol{\Gamma}\}$
- $\mathcal{E} = \{E_x = (X = x) : x \in \mathbf{X}\}$

$$\lambda(X = x | \Theta = \theta, \Gamma = \gamma) = \begin{cases} \left[\prod_{i=1}^{k} \binom{\theta}{x_i}\right] \gamma^{||x||_1} (1-\gamma)^{\theta k - ||x||_1}, & \text{if } \theta \geq ||x||_\infty, \\ 0 & \text{otherwise,} \end{cases}$$

# Example: Nuisance parameter elimination (2)

Take:

- $\mathcal{A}_{\mathcal{L}} = \langle \mathcal{L} \rangle^*$ and $\mathcal{A}_{\mathcal{E}} = \langle \mathcal{E} \rangle$
- $\varphi$, vacuous belief ($\varphi(\Omega) = 1$ and 0 otherwise) on $\mathcal{A}_{\mathcal{L}}$ giving rise to the class

  $$\mathcal{P}^{\mathbf{p}} = \{\tilde{\pi} \,:\, \text{conditional prior on } \mathcal{A}_{\mathcal{L}} \times \mathcal{A}^0_{\mathcal{L}}\}$$

  whose upper envelope $\overline{\pi}^{\mathbf{p}} = \max \mathcal{P}^{\mathbf{p}}$ is defined for $F|K \in \mathcal{A}_{\mathcal{L}} \times \mathcal{A}^0_{\mathcal{L}}$ as

  $$\overline{\pi}^{\mathbf{p}}(F|K) = \begin{cases} 1 & \text{if } K \subseteq F, \\ 0 & \text{otherwise,} \end{cases}$$

## GOAL

Make inference on conditional events $(X = x | (\Theta, \Gamma) \in \{\theta\} \times \mathbf{\Gamma})$

$\Longrightarrow$ The profile likelihood is a supremum of integrated likelihoods

$$\begin{aligned} \hat{\lambda}(X = x | \Theta = \theta) &= \overline{P}^{\mathbf{fd}}_{\varphi}(X = x | (\Theta, \Gamma) \in \{\theta\} \times \mathbf{\Gamma}) \\ &= \oint \lambda(X = x | \Theta = \theta, \Gamma = \gamma) \overline{\pi}^{\mathbf{p}}(\mathrm{d}(\Gamma = \gamma) | \Theta = \theta) \\ &= \sup_{\gamma} \lambda(X = x | \Theta = \theta, \Gamma = \gamma) \end{aligned}$$

# Conclusions

We consider Bayesian inference under a precise strategy $\sigma$ and ambiguity in the prior information through a full B-conditional belief function $Bel_B$: a characterization for the envelopes of the class of full conditional probabilities dominating the assessment $\{Bel_B, \sigma\}$ is provided.

Future research: to introduce ambiguity also in the strategy by considering an imprecise strategy $\beta$ such that $\beta(\cdot|H_i)$ is a belief function, for every $H_i \in \mathcal{L}$, possibly removing the finiteness assumption. This would lead to a theory to compare with that of Walley[8].

[8] Miranda-Zaffalon 2013, 2017