Imprecise Swing Weighting for Multi-Attribute Utility Elicitation Based on Partial Preferences

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The Crab of Imprecision

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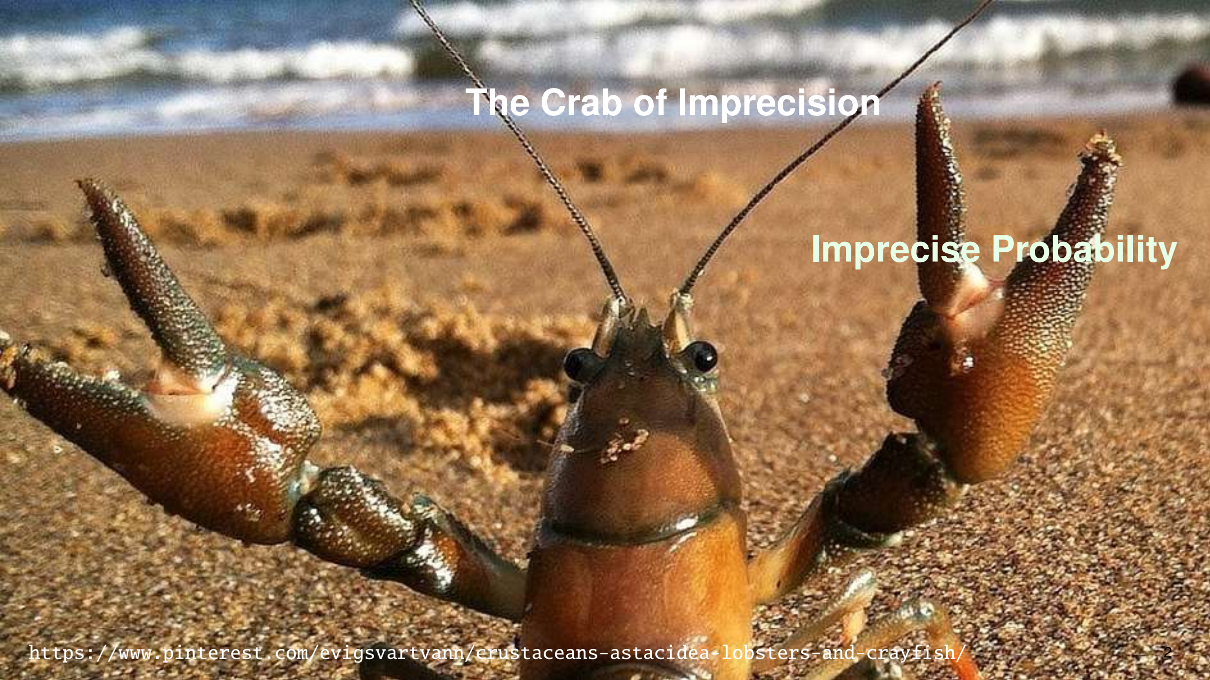
The Crab of Imprecision



Imprecise Probability

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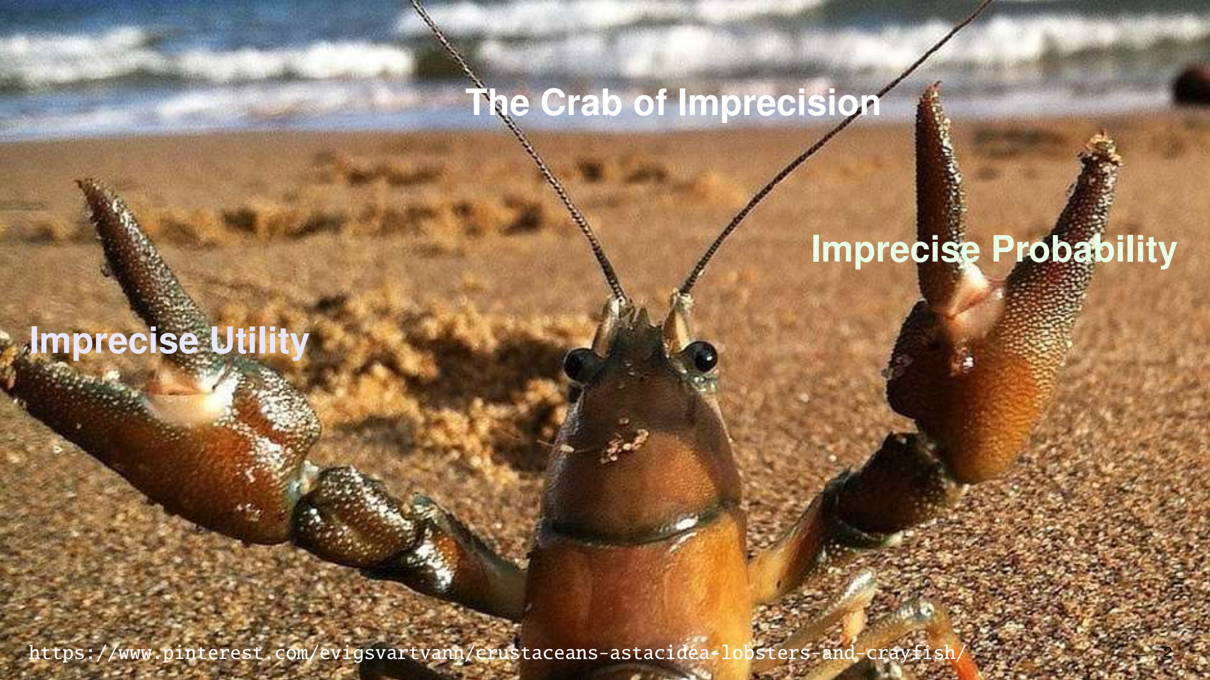
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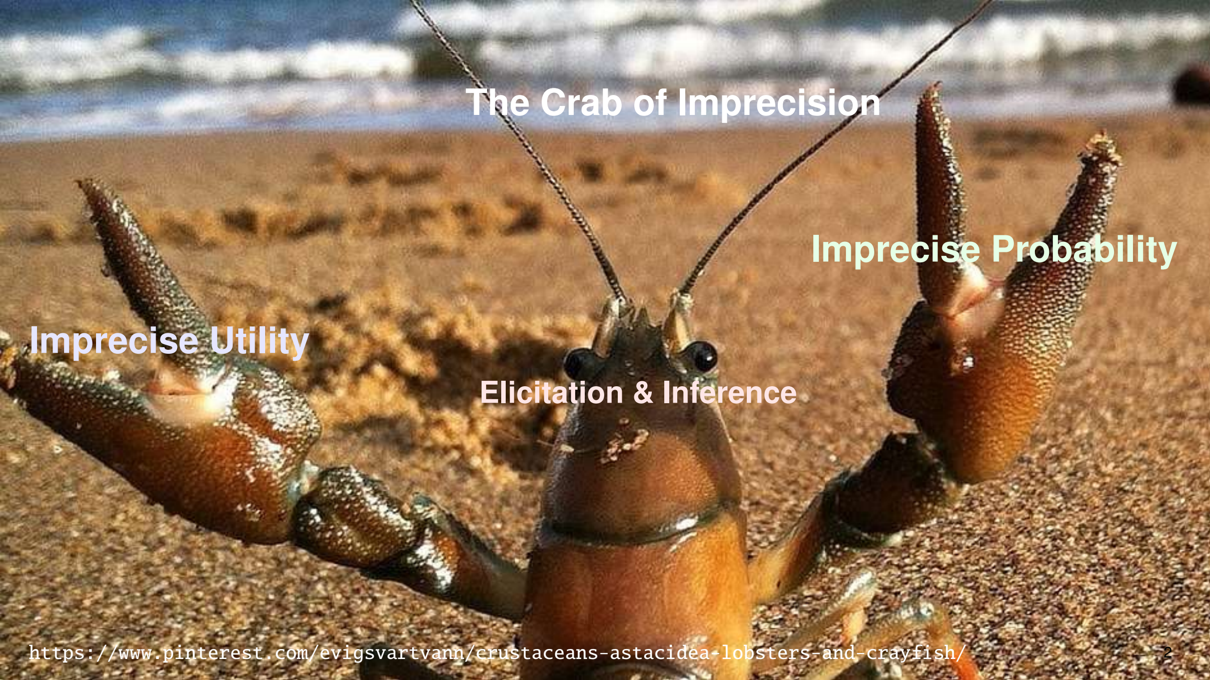
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Elicitation & Inference
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Outline

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Assumptions

Main Contributions & Results

Application

Conclusion
- Main Messages
- Further Reading

Assumptions

Rewards

Each reward $r = (a_1, \ldots, a_n)$ comprises of n attributes. Set of rewards $\mathcal{R} := \mathcal{A}_1 \times \cdots \times \mathcal{A}_n$.

Lotteries

A *lottery* ℓ on $\mathcal{R}$ = probability mass function over $\mathcal{R}$. $L(\mathcal{R})$ = set of all lotteries over $\mathcal{R}$.

Utility Function

…on $\mathcal{R}$ = any function $U \colon \mathcal{R} \to \mathbb{R}$. Lifted to $L(\mathcal{R})$ in the usual way: $U(\ell) := \sum_{r \in \mathcal{R}} \ell(r) U(r)$

Preferences

We assume that our preferences form a preorder $\succeq$ on $L(\mathcal{R})$ and can be represented through a *set* $\mathcal{U}$ of utility functions $U \colon L(\mathcal{R}) \to \mathbb{R}$:

$$\ell_1 \succeq \ell_2 \iff \forall U \in \mathcal{U} \colon U(\ell_1) \geq U(\ell_2)$$

Procedure for identifying $\mathcal{U}$?

Assumptions

Additive Form

$$U(a_1, \dots, a_n) = \sum_{i=1}^{n} k_i U_i(a_i)$$

Marginal Utility Functions

$U_1, \dots, U_n$ assumed to be known precisely.

Attribute Weights

$k_1, \dots, k_n$ not assumed to be known.

Procedure for identifying a set of attribute weights?

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Contribution 1: General Method for Eliciting Bounds on Weights

Elicitation

(i) Consider *any* joint rewards $r_0, \dots, r_n$ for which we have that $r_0 \preceq r_j \preceq r_n$

(ii) Find largest $\underline{\alpha}_j$ and smallest $\overline{\alpha}_j$ so that $(1-\underline{\alpha}_j)r_0 \oplus \underline{\alpha}_j r_n \preceq r_j \preceq (1-\overline{\alpha}_j)r_0 \oplus \overline{\alpha}_j r_n$

1. Generalises classical swing weighting
2. Generalises various imprecise methods from literature (Mustajoki 2005, Riabacke 2009, Gomes 2011, ...)
3. Clean operational interpretation which is not stated in literature (as far as I know)

Contribution 2: Linear Constraint Representation

Notation

With $r_j = (a_{j1}, \ldots, a_{jn})$, let $u_j := (U_1(a_{j1}), \ldots, U_n(a_{jn}))$. Let $k := (k_1, \ldots, k_n)$.

Inference Theorem

Stated preferences lead to a set of linear inequalities on weights $(k_1, \ldots, k_n)$:

$$\forall j \in \{1, \ldots, n-1\}: (u_j - (1-\underline{\alpha}_j)u_0 - \underline{\alpha}_j u_n) \cdot k \geq 0 \geq (u_j - (1-\overline{\alpha}_j)u_0 - \overline{\alpha}_j u_n) \cdot k$$

$$1 \cdot k = 1$$

Contribution 3: Consistency and Uniqueness

What is Consistency & Uniqueness?

1. Existence of a solution for all possible choices of $0 \leq \underline{\alpha}_j \leq \overline{\alpha}_j \leq 1$
2. Uniqueness of solution when $\underline{\alpha}_j = \overline{\alpha}_j$ for all j

Uniqueness Theorem

Assume that u_0 is constant, and that the vectors $(u_1, \ldots, u_{n-1}, 1)$ are linearly independent. Let λ_j be the coefficients that decompose u_n as a linear combination of $(u_1, \ldots, u_{n-1}, 1)$:

$$u_n = \lambda_n + \sum_{j=1}^{n-1} \lambda_j u_j$$

Then the precise case has a unique solution if and only if $\sum_{j=1}^{n-1} \alpha_j \lambda_j \neq 1$.

Consistency Theorem

When $\lambda_1 \leq 0, \ldots, \lambda_{n-1} \leq 0$, then solution exists for all possible choices of $0 \leq \underline{\alpha}_j \leq \overline{\alpha}_j \leq 1$.

Outline

Application: Marmorkrebs

What is Marmorkrebs?

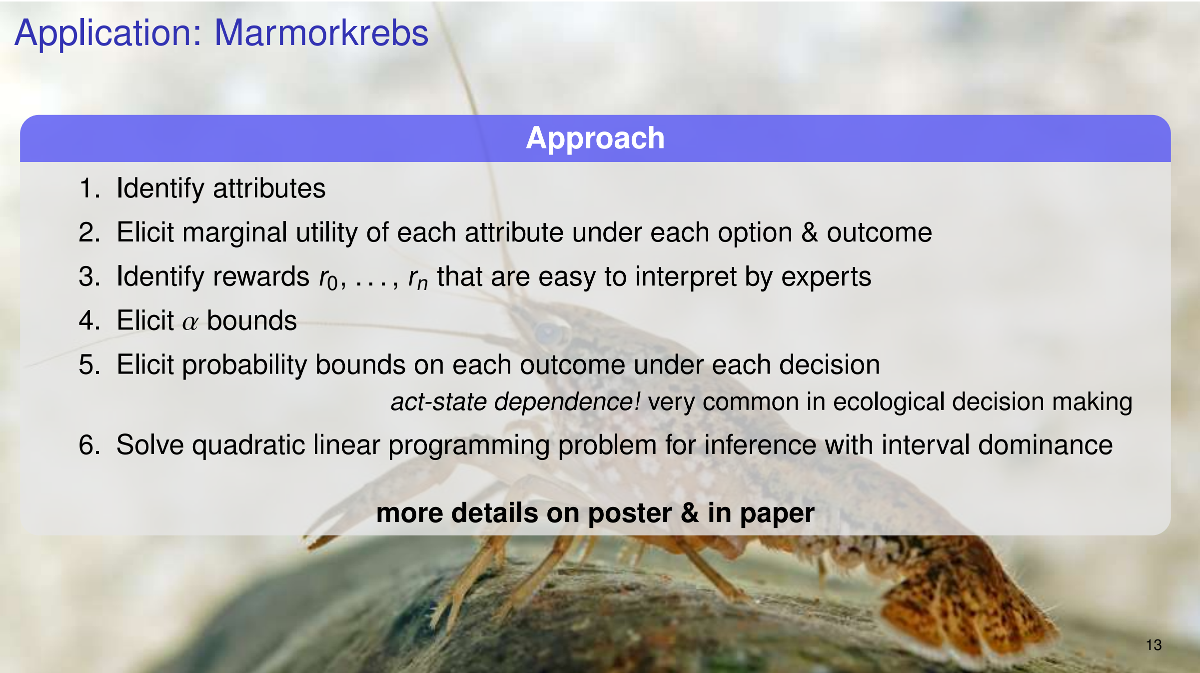
Origin unknown, first known individuals from pet trade 1990’s.
Can reproduce asexually, high reproduction rate, damages ecosystems.

Ecological Decision Problem

Eradicate invasive marmorkrebs recently observed in a lake

Options

(I) Do nothing
(II) Mechanical removal
(III) Drain system and remove individuals by hand
(IV) Drain system, dredge and sieve to remove individuals
(V) Decomposable biocide plus drainage
(VI) Increase pH plus drainage and removal by hand



Application: Marmorkrebs

Approach

1. Identify attributes
2. Elicit marginal utility of each attribute under each option & outcome
3. Identify rewards $r_0, \ldots, r_n$ that are easy to interpret by experts
4. Elicit α bounds
5. Elicit probability bounds on each outcome under each decision

   act-state dependence! very common in ecological decision making
6. Solve quadratic linear programming problem for inference with interval dominance

more details on poster & in paper

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Conclusion: Main Messages

Extremely Flexible Generalisation of Swing Weighting

(i) Operational

(ii) Partial preferences when attributes are hard to weigh

(iii) Flexible choice of rewards to match expert experience

(iv) Strong consistency & uniqueness properties

Demonstrated Benefits of Imprecision in Ecological Decision Making

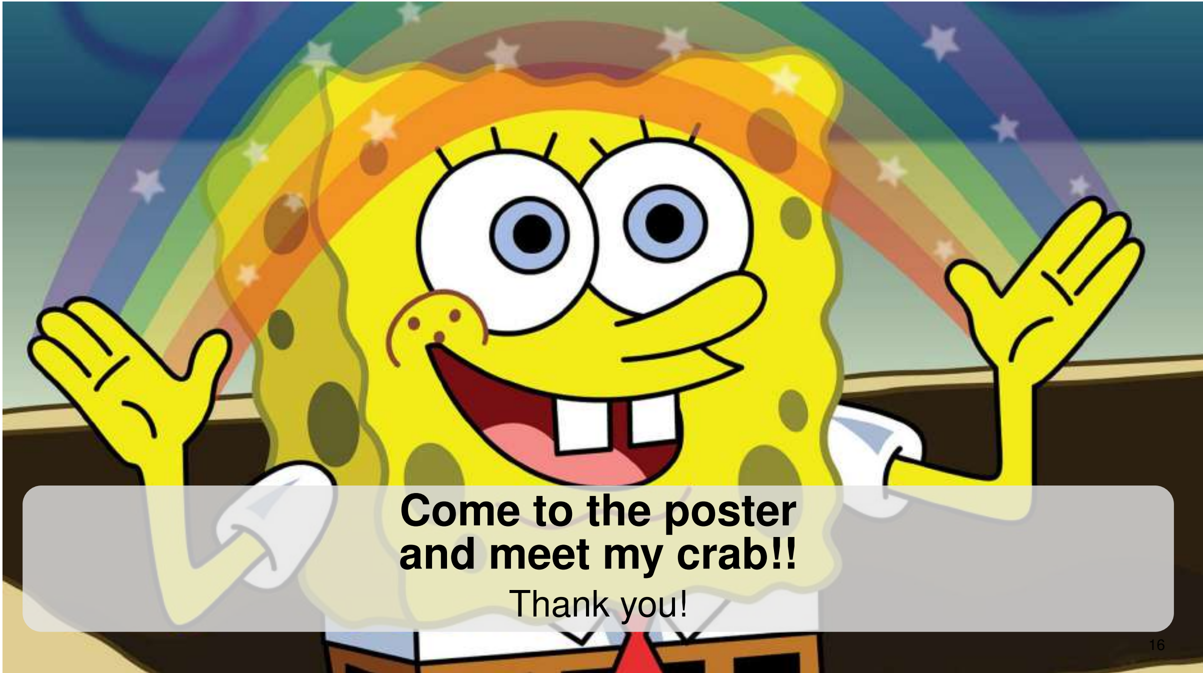
(i) Value ambiguity expressed through direct comparison of simple lotteries

(ii) Uncertainty about success more reliably incorporated with intervals

(iii) Realm of quadratic programming

(iv) Act-state dependence means interval dominance

(v) High imprecision in inputs does not need to imply vacuous results



Come to the poster and meet my crab!!

Thank you!

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