# Dependence among probability boxes in fault trees

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## Poster Abstract

We consider the simple problem of projecting p-boxes in fault trees, for which there are many real practical uses, and we restrict ourselves to only forward uncertainty propagation problems, in which the characterizations of the probabilities of the leaf events are projected up the tree to evaluate the probability of the top event. Boole first considered the problem of projecting probabilities through the logical operations of AND and OR. Although his treatment is often criticized as cavalier or even wrong, he did realize the consequence of making no assumption about the dependence among the events, and he presaged the inequalities that modern probabilists attribute to Fréchet. Hailperin generalized and extended Boole's ideas to consider interval-valued estimates of probabilities in Boolean expressions, which are equivalent to fault trees, and showed that the necessary calculations to find optimal solutions generally require linear programming. This uncertainty logic was reconceived by Kozine and Filimonov in terms of coherent lower previsions. In this theory, the lower prevision of a binary event $X$ is its lower probability $\underline{P}(X)$ defined as real-valued, so the characterization of the uncertain logical value is via an interval, namely $[\underline{P}(X), \overline{P}(X)]$, where $\overline{P}(X) = 1 - \underline{P}(\text{NOT } X)$.

Introducing *distributions* or *sets of distributions* into fault trees as characterizations of leaf nodes creates subtleties in how dependencies should be handled that were not encountered in the interval-probability approaches of Hailperin or Kozine and Filimonov. For instance, assuming that the modeled leaf events are stochastically independent does not guarantee that the information sets about the probabilities associated with those events are also independent. If there are two levels at which dependence is a concern: among the events, and among the data sets used to characterize them, one can have different assumptions about dependence at each level, and one will get different answers for each combination of assumptions.

We argue that a particular variant of probabilistic logic is an uncertainty calculus appropriate for fault tree analysis in which the events are binary (failed or not-failed) but our information about their failure states is probabilistic and imprecise. We consider five logical operations: AND, OR, NOT, COND (i.e., the probability $P(B|A)$ from marginals $P(A)$ and $P(B)$), and EQUIV (material implication), each under eight models of dependence (no-assumptions, independent, perfect, opposite, mutually exclusive, positive, negative, and Pearson-correlated). This uncertainty calculus generalizes Boolean logic, probability theory (both with and without independence assumptions), and Kleene's strong logic of indeterminacy, which alternative calculi sometime fail to do. For instance, fuzzy logic generalizes Boolean logic, but it does not generalize probabilistic logic, which is usually needed for fault tree analyses. We describe efficient computational algorithms based on simplification strategies from interval analysis and reliability theory, combined with Monte Carlo simulation, that can be used to evaluate many practical fault trees.