(Irrelevant) Natural Extension of Choice Functions

Arthur van Camp ARTHUR.VANCAMP@UGENT.BE
Gert de Cooman GERT.DECOOMAN@UGENT.BE
Ghent University, IDLab
Technologiepark – Zwijnaarde 914, 9052 Zwijnaarde, Belgium

Enrique Miranda MIRANDAENRIQUE@UNIOVI.ES
University of Oviedo, Dep. of Statistics and O.R.
Oviedo, Spain

Abstract

Given a finite possibility space $\mathcal{X}$, a *choice function* is a map that assigns, to any finite set of gambles on $\mathcal{X}$—also called *option set*—those that are considered acceptable to a subject. It generalises sets of desirable gambles, in the sense that the associated preference relation is not necessarily determined by binary comparisons. The rationality of the choices is modelled by the notion of coherence.

The classical theory of coherent choice functions was generalised by Seidenfeld et al. (2010) to deal with situations where two option sets can be incomparable, something arguably more sensible in situations of imprecise information. In Van Camp et al. (2017), we considered a slightly different axiomatisation, so as to include the optimality criteria of Walley–Sen maximality as a particular case.

One constraint of our model is that we assume that any coherent choice function is defined on the class of all option sets, something not too realistic in practice. Here we remedy the situation somewhat by establishing the notion of *natural extension* for choice functions: it determines the most conservative coherent choice function that extends a given one from an arbitrary domain to the class of all option sets.

In addition, we also consider a structural assessment of irrelevance between two variables X_1 and X_2 taking values in respective sets $\mathcal{X}_1$ and $\mathcal{X}_2$. If our information about these variables is modelled by means of respective coherent choice functions C_1 and C_2 on $\mathcal{X}_1$ and $\mathcal{X}_2$, we call their *irrelevant natural extension* the most conservative coherent choice function on $\mathcal{X}_1 \times \mathcal{X}_2$ that is compatible with C_1 and C_2 and the assumption of irrelevance. Moreover, we compare with the eponymous notion established by de Cooman and Miranda (2012) for sets of desirable gambles.

References

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