Learning Acyclic Directed Mixed Graphs from Observations and Interventions

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Causal effect identification in original ADMGs

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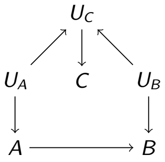	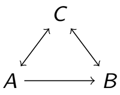	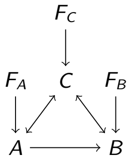

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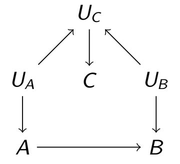	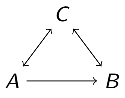	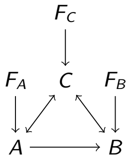

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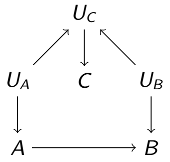	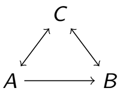	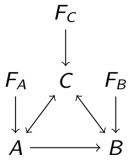

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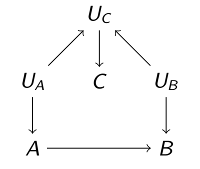	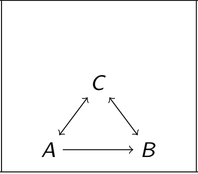	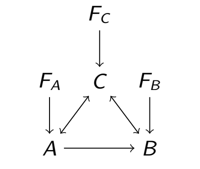

- $X \leftrightarrow Y$ in G iff $U_X \not\perp_D U_Y | \emptyset$.
- Effect identification in G: $p(B|do(A)) = p(B|A)$.
- Identification is possible due to Pearl's *do*-calculus on G':
  - Rule 1 (insertion/deletion of observations):
    $p(Y|do(X), Z \cup W) = p(Y|do(X), W)$ if $Y \perp_{G'} Z | X \cup W || X$.
  - Rule 2 (intervention/observation exchange):
    $p(Y|do(X), do(Z), W) = p(Y|do(X), Z \cup W)$ if $Y \perp_{G'} F_Z | X \cup W \cup Z || X$.
  - Rule 3 (insertion/deletion of interventions):
    $p(Y|do(X), do(Z), W) = p(Y|do(X), W)$ if $Y \perp_{G'} F_Z | X \cup W || X$.

  Where $\cdot \perp_{G'} \cdot | \cdot || X$ denotes separation in G' after intervention on X.

Causal effect identification in **alternative** ADMGs

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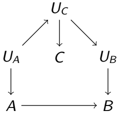
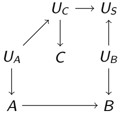
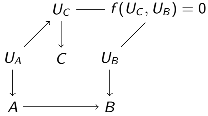
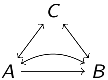
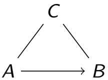
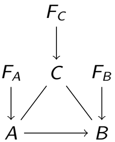
		

Original ADMG G	**Alternative** ADMG H	**Alternative** ADMG H'
		

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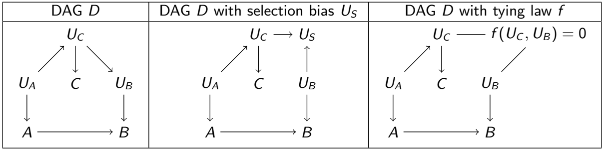

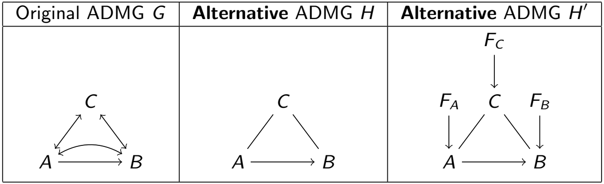

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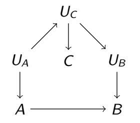
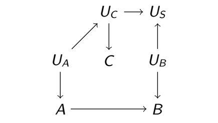
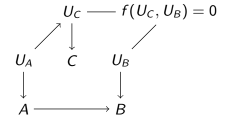
DAG D



DAG D with selection bias U_S



DAG D with tying law f



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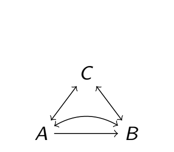
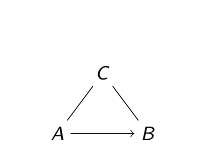
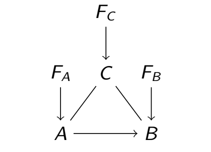


Alternative ADMG H



Alternative ADMG H'



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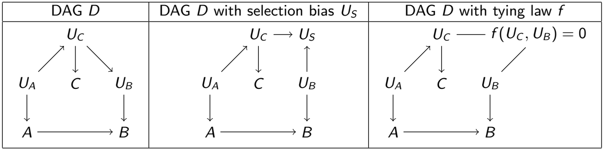

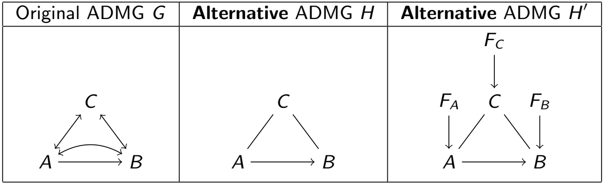

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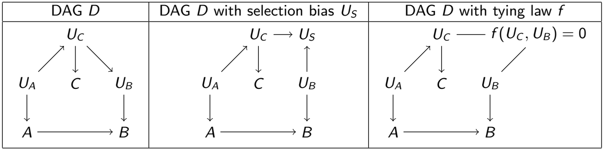

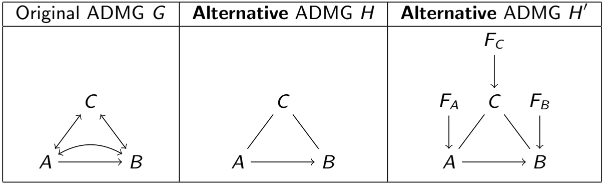

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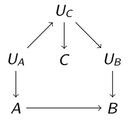	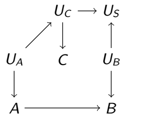	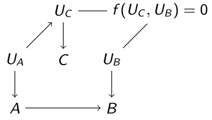

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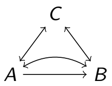	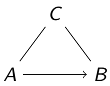	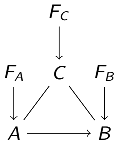

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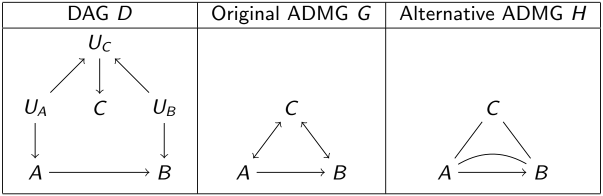
Causal effect identification in alternative ADMGs



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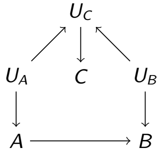	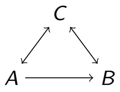	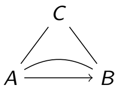

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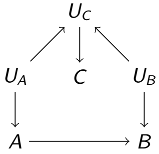	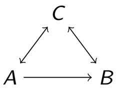	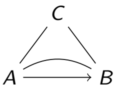

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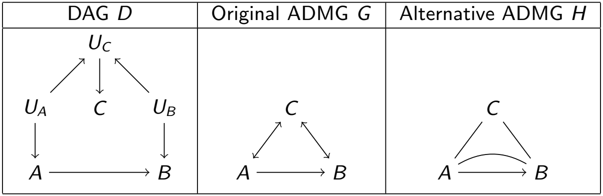

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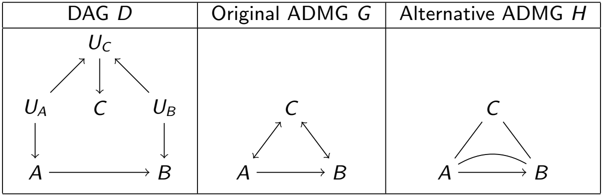

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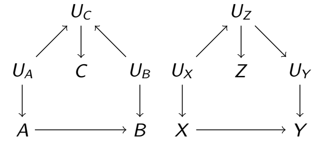	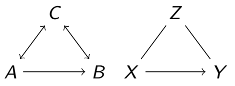

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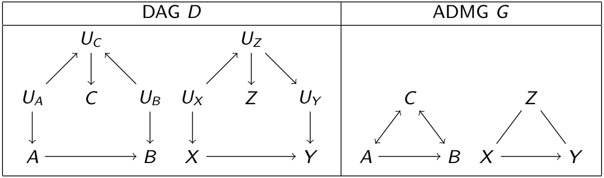

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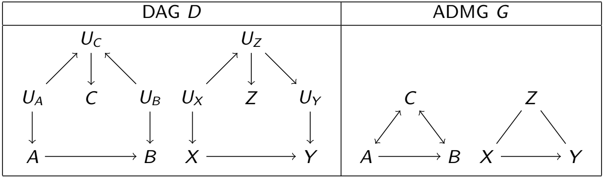

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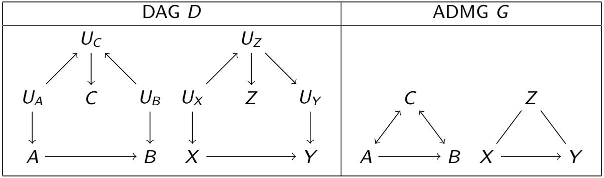

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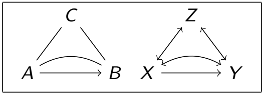

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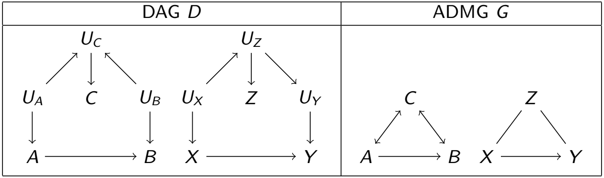

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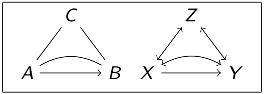

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- Future work includes
  - deriving local and pairwise Markov properties for ADMGs,
  - characterizing Markov equivalence of ADMGs, and
  - deriving factorization induced by ADMGs.

Final Remarks

- We introduce an assumption-free exhaustive algorithm for learning ADMGs from observations and interventions via answer set programming.
- The algorithm can easily be modified to learn original and alternative ADMGs, AMP and MVR CGs, DAGs, UGs and BGs.
- Finally, it is worth mentioning that
  - ADMGs can also be seen as (non-causal) statistical models,
  - no existing family of mixed graphical models subsumes ADMGs, and
  - the definitions of intervention and separation in ADMGs generalize those for original and alternative ADMGs.
- Future work includes
  - deriving local and pairwise Markov properties for ADMGs,
  - characterizing Markov equivalence of ADMGs, and
  - deriving factorization induced by ADMGs.

Thanks for your attention.