A Differential Approach to Causality in Staged Trees

Christiane Görgen and Jim Q. Smith

PGM 2016

Warwick Statistics

This research is funded by EPSRC grant EP/L505110/1.

Causal manipulation on probability trees

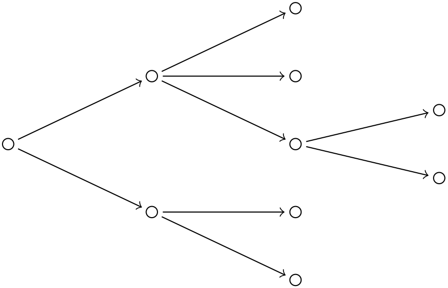
- more general than interventions on DAGs
- easily done in symbolic framework[1]

[1] Adnan Darwiche. A differential approach to inference in Bayesian networks. J. ACM, 50(3):280–305 (electronic), 2003.

Probability trees

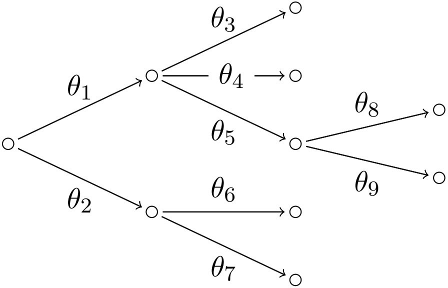
Probability trees

- *event tree* graphs



Probability trees

- *event tree* graphs
- edge labels (probabilities)



Staged probability trees

probability tree + conditional independence assumptions

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Example

Students at Warwick university...

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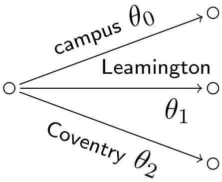

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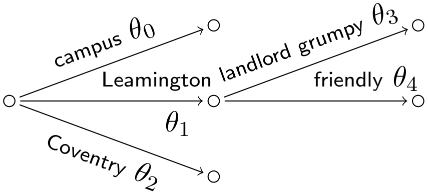

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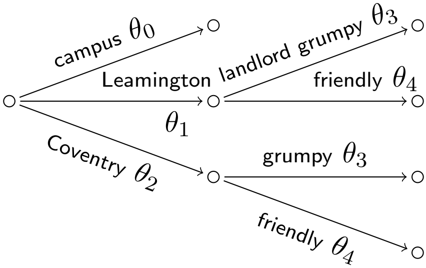

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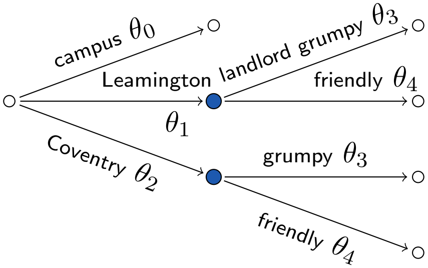

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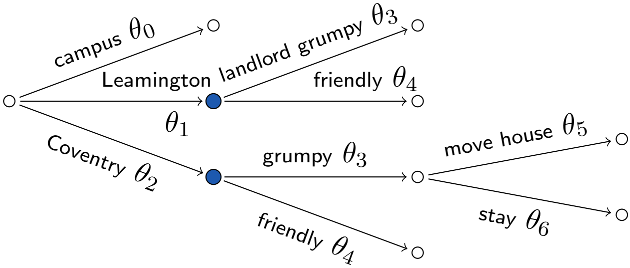

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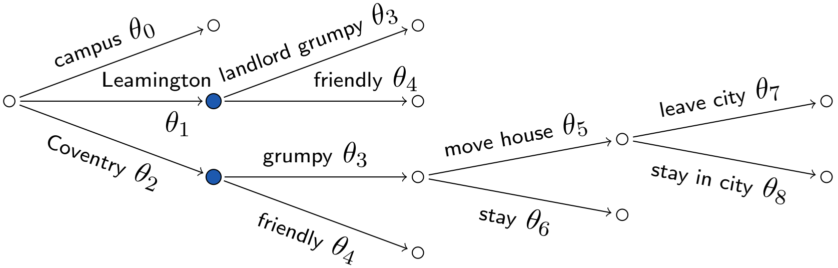
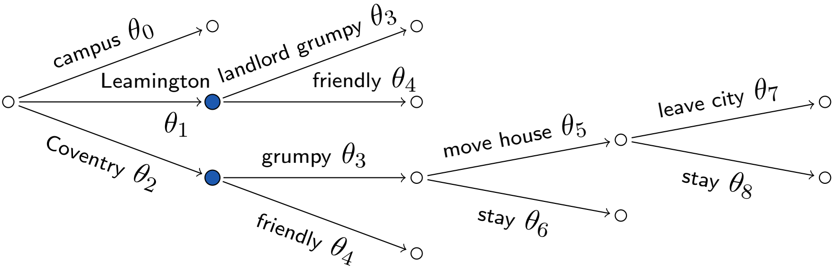


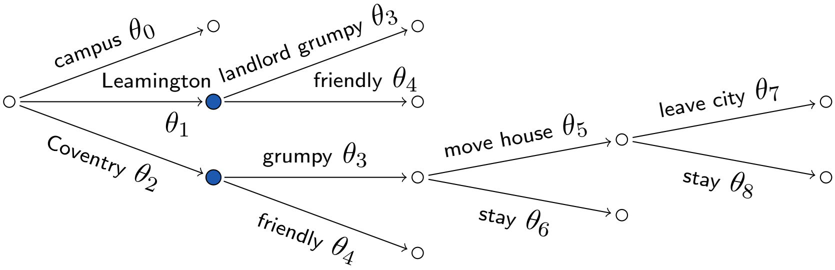
Interventions (graphically)



[2]Peter Thwaites. Causal Identifiability via Chain Event Graphs. Artificial Intelligence, 195:291–315, 2013.

Interventions (graphically)

Impose policy forcing students to live in Coventry:



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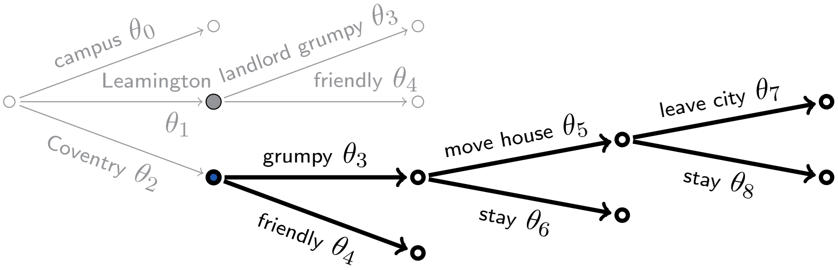

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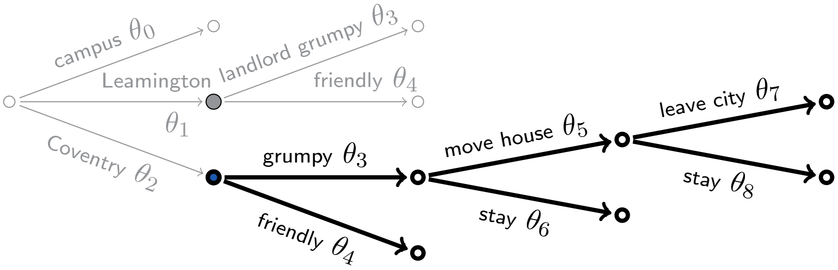

causal interventions[2] $\rightsquigarrow$ projections onto a subtree

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More flexible than DAGs

Staged trees contain discrete Bayesian networks as a special case

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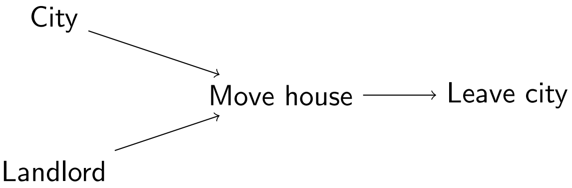

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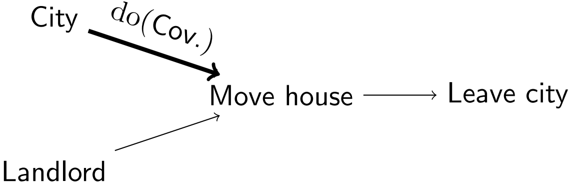

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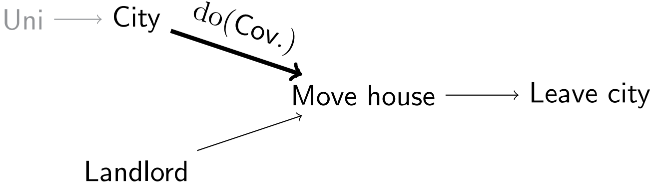

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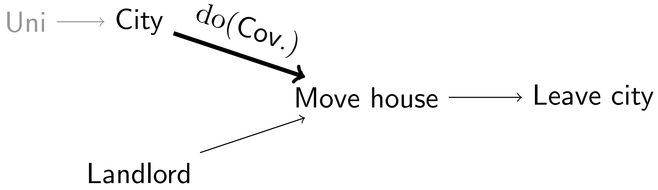

but are more general and more expressive!

Two questions

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- What if we want to do causal manipulations in staged trees without referring to a graph?

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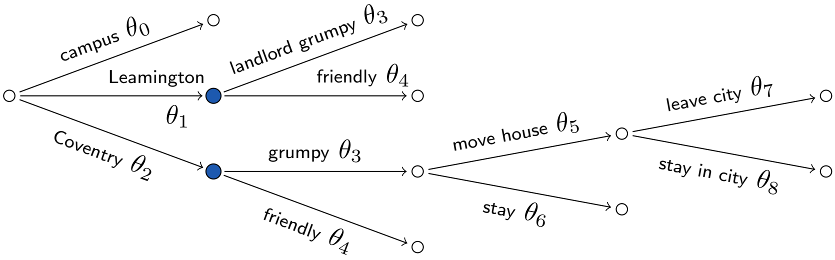
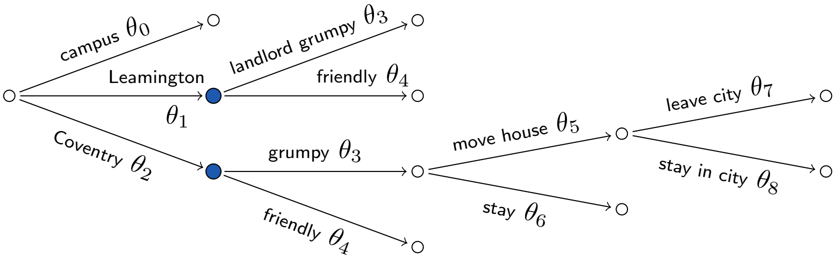
- What if there is a sequence of manipulations we want to perform consecutively?

Replacing the graph by a polynomial



Replacing the graph by a polynomial

Every staged tree is in one-to-one correspondence with a nested polynomial:



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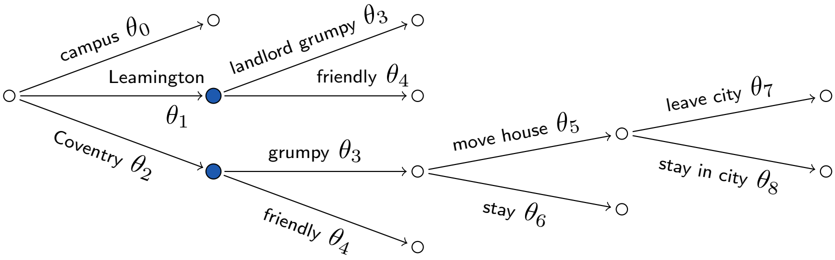

$$c_{\mathcal{T}}(\boldsymbol{\theta}) = \theta_0 + \theta_1\theta_3 + \theta_1\theta_4 + \theta_2\theta_3\theta_5\theta_7 + \theta_2\theta_3\theta_5\theta_8 + \theta_2\theta_3\theta_6 + \theta_2\theta_4$$

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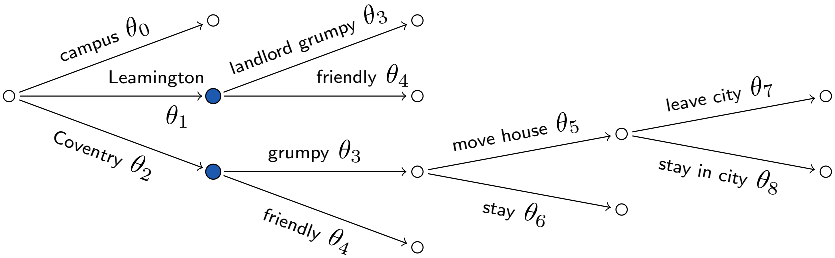

$$
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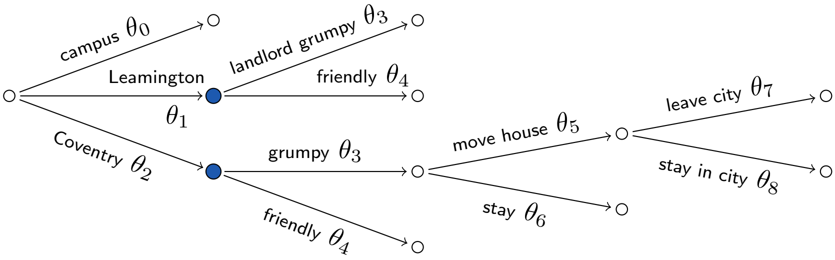
&= \theta_0 + \theta_1(\theta_3 + \theta_4) + \theta_2(\theta_3(\theta_5(\theta_7 + \theta_8) + \theta_6) + \theta_4)
\end{aligned}
$$

Interventions (symbolically)

Manipulate a tree using a **differentiation operation** on this polynomial:



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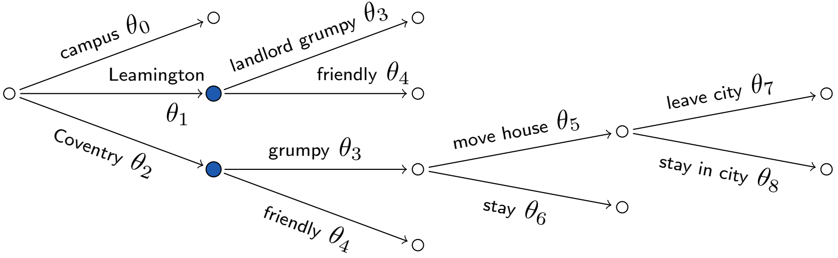

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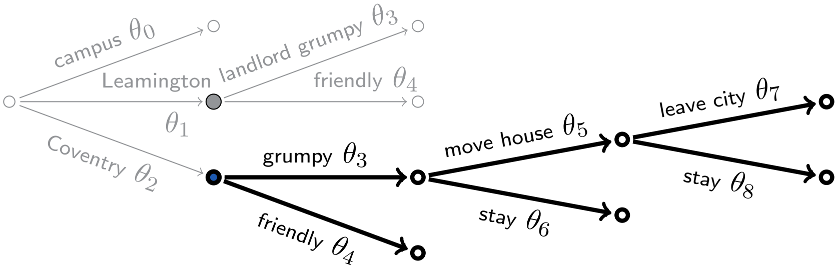

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$$\frac{\partial}{\partial\theta_2} c_{\mathcal{T}}(\boldsymbol{\theta}) = \theta_3\theta_5\theta_7 + \theta_3\theta_5\theta_8 + \theta_3\theta_6 + \theta_4$$

Advantages of the symbolic approach

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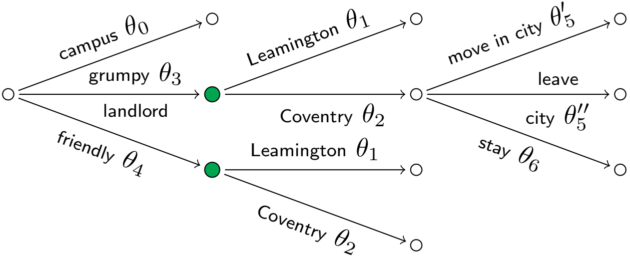

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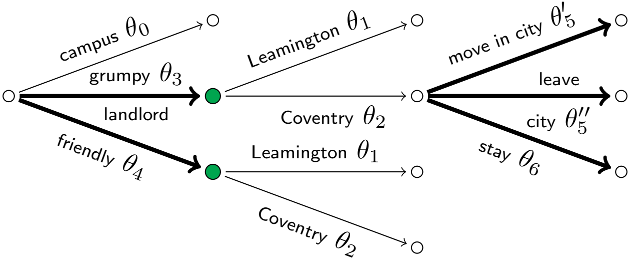

Interventions on the polynomial are more general than vertex manipulations!

Local differentiation operation

Replace a staged tree by a polynomial

$$c_{\mathcal{T}}(\boldsymbol{\theta}) = \sum_{(v_0,v_1)\in E(v_0)} \theta(v_0,v_1) \left(\sum_{(v_1,v_2)\in E(v_1)} \theta(v_1,v_2) \left(\cdots \left(\sum_{(v_{k-1},v_k)\in E(v_{k-1})} \theta(v_{k-1},v_k) \right) \right) \right)$$

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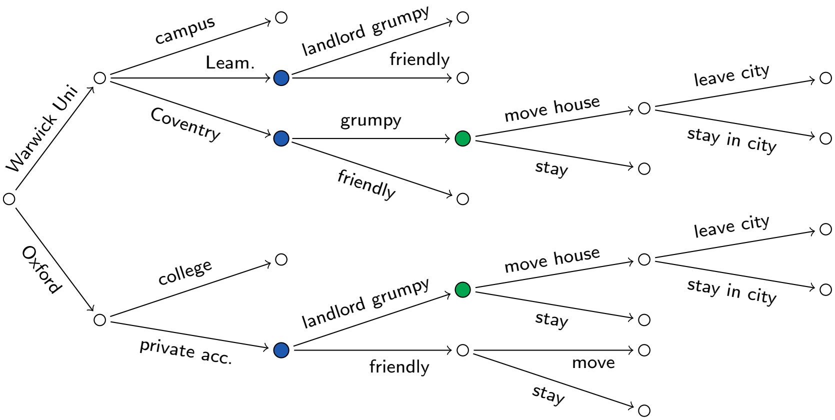
and perform a local differentiation on that:

$$\sum_{(v_0,v_1)\in E(v_0)} \theta(v_0, v_1) \left(\cdots \left(\frac{\partial}{\partial \theta_j^\star} \sum_{(v_{j-1},v_j)\in E(v_j)} \theta(v_{j-1}, v_j) \left(\cdots \left(\sum_{(v_{k-1},v_k)\in E(v_{k-1})} \theta(v_{k-1}, v_k) \right) \right) \right) \right)$$

Example: two interventions



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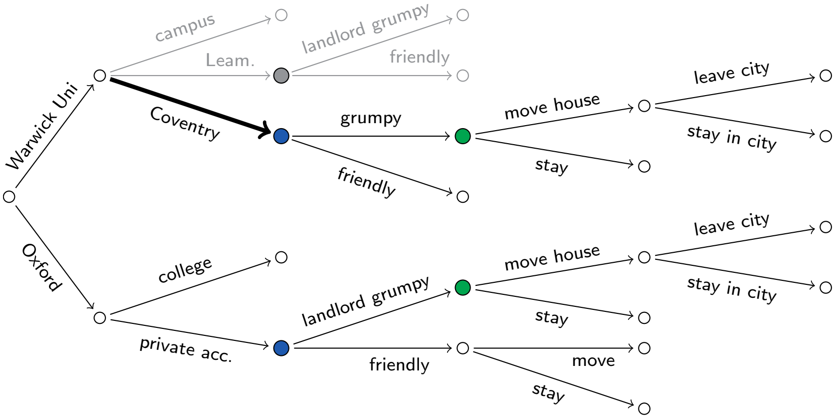
Differentiate locally on florets:



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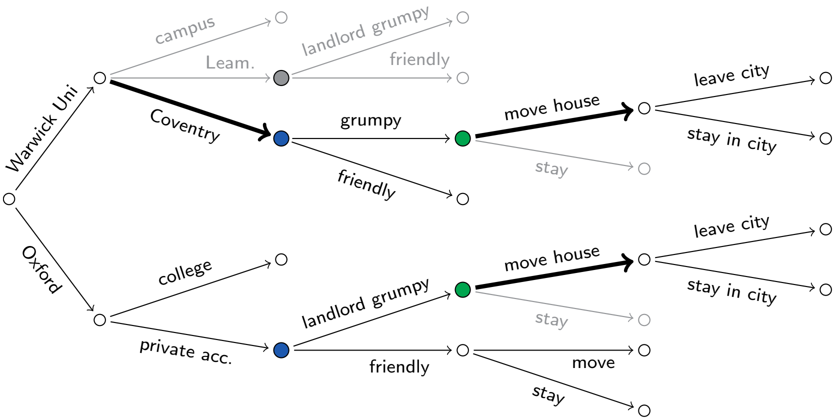

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- does not rely on graphical representation $\rightsquigarrow$ flexible and very general
- used only algebraic description of a parametric model: method can be used in models far more general than staged trees

Thank you very much for your attention!

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Abstract

In this paper, we apply a recently developed differential approach to inference in staged tree models to causal inference. Staged trees generalise modelling techniques established for Bayesian networks (BN). They have the advantage that they can depict highly nuanced structure impossible to express in a BN and also enable us to perform causal manipulations associated with very general types of interventions on the system. Conveniently, what we call the interpolating polynomial of