Measuring Uncertainty in Orthopairs

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Introduction

- Orthopair = pair of disjoint sets $A \cap B = \emptyset$
- Often a bipolar information interpretation: A = positive, B = negative
- Appear in several contexts. Examples are:
  - Representing partial knowledge
  - Trust and distrust (in Social Network Analysis)
  - Rough sets
- Aim: define uncertainty measures (notice that $A \cup B \neq U$)
  - in a single orthopair
  - in a collection of orthopairs
- Uncertainty = Lack of Knowledge? Fuzziness? Contradictions?
- Take inspiration from: IFS, Fuzzy Sets, Rough Sets, Possibility Theory

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Outline

1. *Orthopairs*
2. *Uncertainty in a Single Orthopair*
   - From IFS
   - From Fuzzy Sets
3. *Uncertainty in a Collection of Orthopairs*

Definitions

- X = universe, (P, N) orthopair, $P \cap N = \emptyset$
- $Bnd = X \setminus (P \cup N)$ boundary
- $O(X)$ collection of all orthopairs on X
- $O(X)$ is in bijection with three-valued sets $f_o : X \mapsto \{0, \frac{1}{2}, 1\}$:

$$f_o(x) = \begin{cases} 1 & \text{if } x \in P \\ 0 & \text{if } x \in N \\ \frac{1}{2} & \text{otherwise} \end{cases}$$

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Pointwise Ordering

Order on V	Order on $O(X)$	Symbol	Type
$0 \leq \frac{1}{2} \leq 1$	$P_1 \subseteq P_2, N_2 \subseteq N_1$	$\leq_t$	Total
$\frac{1}{2} \leq 1 \leq 0$	$N_1 \subseteq N_2, Bnd_2 \subseteq Bnd_1$	$\leq_N$	Total
$\frac{1}{2} \leq 0 \leq 1$	$P_1 \subseteq P_2, Bnd_2 \subseteq Bnd_1$	$\leq_P$	Total
$\frac{1}{2} \leq 1, \frac{1}{2} \leq 0$	$P_1 \subseteq P_2, N_1 \subseteq N_2$	$\leq_I$	Partial
$0 \leq \frac{1}{2}, 0 \leq 1$	$P_1 \subseteq P_2, Bnd_1 \subseteq Bnd_2$	$\leq_{PB}$	Partial
$1 \leq \frac{1}{2}, 1 \leq 0$	$N_1 \subseteq N_2, Bnd_1 \subseteq Bnd_2$	$\leq_{NB}$	Partial

- $\leq_t$ truth ordering, $\leq_I$ knowledge ordering
- other non-pointwise ordering exist [Granular Computing, 1, 2016]

Aggregation operations

From the three total order we derive:

- Strong Kleene meet and join

$$(P_1, N_1) \sqcap_t (P_2, N_2) := (P_1 \cap P_2, N_1 \cup N_2)$$
$$(P_1, N_1) \sqcup_t (P_2, N_2) := (P_1 \cup P_2, N_1 \cap N_2)$$

- Weak Kleene meet and join

$$(P_1, N_1) \sqcap_P (P_2, N_2) := (P_1 \cap P_2, (N_1 \cap N_2) \cup [(N_1 \cap P_2) \cup (N_2 \cap P_1)])$$
$$(P_1, N_1) \sqcap_N (P_2, N_2) := ((P_1 \cap P_2) \cup [(P_1 \cap N_2) \cup (P_2 \cap N_1)], N_1 \cap N_2)$$

- Sobocinski meet and join

$$(P_1, N_1) \sqcup_N (P_2, N_2) := (P_1 \setminus N_2 \cup P_2 \setminus N_1, N_1 \cup N_2)$$
$$(P_1, N_1) \sqcup_P (P_2, N_2) := (P_1 \cup P_2, N_1 \setminus P_2 \cup N_2 \setminus P_1)$$

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- Sobocinski meet and join

$$(P_1, N_1) \sqcup_N (P_2, N_2) := (P_1 \backslash N_2 \cup P_2 \backslash N_1, N_1 \cup N_2)$$
$$(P_1, N_1) \sqcup_P (P_2, N_2) := (P_1 \cup P_2, N_1 \backslash P_2 \cup N_2 \backslash P_1)$$

Aggregation operations

- Pessimistic combination operator (the meet from the knowledge ordering $\preceq_I$)

$$(P_1, N_1) \sqcap_I (P_2, N_2) := (P_1 \cap P_2, N_1 \cap N_2)$$

- Optimistic combination operator (when definable, the join from $\preceq_I$)

$$(P_1, N_1) \sqcup_I (P_2, N_2) := (P_1 \cup P_2, N_1 \cup N_2)$$

- Consensus

$$O_1 \odot O_2 := (P_1 \setminus N_2 \cup P_2 \setminus N_1, N_1 \setminus P_2 \cup N_2 \setminus P_1)$$

Reconcile two orthopairs by keeping as positive part only what is not considered negative by the other and dually for the negative part

Generalization

- Intuitionistic Fuzzy Sets (IFS)
  IFSs are pairs of fuzzy sets $f_P, f_N : X \mapsto [0,1]$ such that for all $x \in X$, $f_P(x) + f_N(x) \leq 1$
- (Boolean) Possibility Theory
  - An Orthopair coincides with the particular class of hyper-rectangular Boolean possibility distributions on the space $\{0,1\}^n$
  - A generic Boolean possibility distribution π, corresponds to a set of orthopairs

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A simple measure

A central role will be played by the counting measure

$$E_O((P,N)) = \frac{|Bnd|}{|X|}$$

- Counting the *uncertain* objects in the boundary
- Also named *roughness* in rough set theory

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IFS Entropy measure

Different axiomatization exists

- To characterize fuzziness [Szmidt and Kacprzyk, 2001]
- To characterize lack of knowledge [Pal et al, 2013]

Results

1. On orthopairs the two axiomatization coincide
2. E_O is the only function (up to constants) to satisfy the axioms

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Entropy that reduces to E_O

Zhang, 2013: survey of entropy measures

- $E_{BB}(O) = \frac{1}{|X|}\sum_{x\in X}\chi_{Bnd_O}(x)$
- $E_{SK}(O) = \frac{1}{|X|}\sum_{x\in X}\frac{min(\chi_{P_O}(x),\chi_{N_O}(x))+\chi_{Bnd_O}(x)}{max(\chi_{P_O}(x),\chi_{N_O}(x))+\chi_{Bnd_O}(x)}$
- $E_{ZL}(O) = 1 - \frac{1}{|X|}\sum_{x\in X}|\chi_{P_O}(x) - \chi_{N_O}(x)| = E_O(O);$
- $E_{VS}(O) = -\frac{1}{|X|ln2}\sum_{x\in X}[\chi_{P_O}(x)ln\chi_{P_O}(x) + \chi_{N_O}(x)ln\chi_{N_O}(x) - (1 - \chi_{Bnd_O}(x))ln(1-\chi_{Bnd_O}(x)) - \chi_{Bnd_O}(x)ln2]$
- $E_{Y1}(O) = \frac{1}{|X|}\sum_{x\in X}\{\{sin[\frac{\pi}{4}(1+\chi_{P_O}(x)-\chi_{N_O}(x))] + sin[\frac{\pi}{4}(1-\chi_{P_O}(x)+\chi_{N_O}(x))] - 1\}\frac{1}{\sqrt{2}-1}\}$
- $E_{Y2}(O) = \frac{1}{|X|}\sum_{x\in X}\{\{cos[\frac{\pi}{4}(1+\chi_{P_O}(x)-\chi_{N_O}(x))] + cos[\frac{\pi}{4}(1-\chi_{P_O}(x)+\chi_{N_O}(x))] - 1\}\frac{1}{\sqrt{2}-1}\}$
- $E(O) = 1 - \frac{1}{|X|}\sum_{x\in X}[\sqrt{2(\chi_{P_O}(x)-0.5)^2 + 2(\chi_{N_O}(x)-0.5)^2} - \chi_{Bnd_O}(x)]$

Entropy that reduces to 0

- $E_{ZJ}(O) = \frac{1}{|X|} \sum_{x \in X} \frac{min(\chi_{P_O}(x), \chi_{N_O}(x))}{max(\chi_{P_O}(x), \chi_{N_O}(x))}$
- $E_{Z1}(O) = 1 - \sqrt{\frac{2}{|X|} \sum_{x \in X} [(\chi_{P_O}(x) - 0.5)^2 + (\chi_{N_O}(x) - 0.5)^2]}$
- $E_{Z2}(O) = 1 - \frac{1}{|X|} \sum_{x \in X} [|\chi_{P_O}(x) - 0.5| + |\chi_{N_O}(x) - 0.5|]$
- $E_{Z3}(O) = 1 - \frac{2}{|X|} \sum_{x \in X} max(|\chi_{P_O}(x) - 0.5|, |\chi_{N_O}(x) - 0.5|)$
- $E_{Z4}(O) = 1 - \sqrt{\frac{4}{|X|} \sum_{x \in X} max(|\chi_{P_O}(x) - 0.5|^2, |\chi_{N_O}(x) - 0.5|^2)}$
- $E^2_{hc}(O) = \frac{1}{|X|} \sum_{x \in X} [1 - \chi_{P_O}(x)^2 - \chi_{N_O}(x)^2 - \chi_{Bnd_O}(x)^2]$
- $E^{1/2}_r(O) = \frac{2}{|X|} \sum_{x \in X} ln[\chi_{P_O}(x)^{1/2} - \chi_{N_O}(x)^{1/2} - \chi_{Bnd_O}(x)^{1/2}]$

Other Measures in IFS

- Knowledge measure $\mathcal{K}$ [Guo, 2016]
  On orthopairs: $\mathcal{K}(O) = 1 - E_O(O)$
- Non specificity H
  On orthopairs

$$H_{IFS}(O) = \begin{cases} \log|U| & \text{if } P = \emptyset \\ \log|P_O \cup Bnd_O| & \text{otherwise} \end{cases}$$

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Measures from Fuzzy Sets

- Entropy [De Luca, Termini, 1972]
  On orthopairs it coincides with E_O
- Non-specificity [Klir, 2004]
  - $H_{Klir}(O) = \frac{1}{2} log|P \cup Bnd| + \frac{1}{2} log|P|\ (P \neq \emptyset)$
  - Different from the IFS based on: $H_{IFS}(O) = \log |P \cup Bnd|$
  - $H_{Klir}(O) \leq H_{IFS}(O)$

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E_O and Aggregation Operations

- Order relations
  - anti-tonic w.r.t. the orders $\leq_I$ (Knowledge Ordering), $\leq_N$ and $\leq_P$
  - *isotonic* w.r.t. the orders $\leq_{NB}$, $\leq_{PB}$
  - non-monotonic w.r.t. the truth ordering $\leq_t$
- Aggregation operators: from knowledge ordering
  - $E_O(O_1), E_O(O_2) \leq E_O(O_1 \sqcap_I O_2) \leq E_O(O_1) + E_O(O_2) + \frac{|P_1 \cap N_2|}{|U|} + \frac{|P_2 \cap N_1|}{|U|}$
  - In case of no conflict
    $E_O(O_1 \sqcap_I O_2) \leq E_O(O_1) + E_O(O_2)$

    $E_O(O_1 \sqcup_I O_2) \leq min(E_O(O_1), E_O(O_2))$
- Aggregation operators: weak/strong Kleene, Sobocinski

$$E_O(O_1 * O_2) \leq E_O(O_1) + E_O(O_2)$$

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General Approach

- A collection of orthopairs $\mathcal{O}$, a probability distribution $P_{\mathcal{O}}$
- A global uncertainty

$$E(\mathcal{O}) = \sum_{O_i \in \mathcal{O}} P(O_i) E_O(O_i)$$

- If we assume equiprobability and E_O the counting measure $E(\mathcal{O}) = \frac{1}{n|U|} \sum_{i=1}^{n} |Bnd_i|$
- Other possibilities (in the paper)
  - Consider an orthopair as a rough set $\rightarrow$ entropy in rough set case
  - Consider $\mathcal{O}$ as representing a possibility distribution

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Measuring conflict

Aim: measure how much the orthopairs overlap

- Two orthopairs: $C(O_1, O_2) = \frac{|P_1 \cap N_2| + |P_2 \cap N_1|}{|X|}$
- More than two
  1. A straightforward solution

$$C_1(\mathcal{O}) = \frac{|x \in X \ s.t. \ \exists (O_i, O_j) \, x \in P_i \cap N_j \text{ or } x \in P_j \cap N_i|}{|X|}$$

  2. A more precise solution: $C_2(\mathcal{O}) = \frac{\sum_x C(\mathcal{O}, x)}{|X|}$

$$C(\mathcal{O}, x) = \frac{|(O_i, O_j) \ s.t. \ x \in P_i \cap N_j \text{ or } x \in P_j \cap N_i|}{(n/2)^2}$$

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Example

- First collection $\mathcal{O}_1$
  $O_1 = (\{1\}, \emptyset)$, $O_2 = (\{1,3\}, \{2\})$, $O_3 = (\{1,2\}, \{4\})$,
  $O_4 = (\{2\}, \{1\})$
- Second collection $\mathcal{O}_2$
  $O'_1 = (\{1\}, \{2,4\})$, $O'_2 = (\{1,3\}, \{2\})$, $O'_3 = (\{2\}, \{1,4\})$,
  $O'_4 = (\{2\}, \{1\})$
- Intuition: $C(\mathcal{O}_1) \leq C(\mathcal{O}_2)$
- However:
  $C_1(\mathcal{O}_1) = C_1(\mathcal{O}_2) = \frac{1}{2}$
  $\frac{5}{16} = C_2(\mathcal{O}_1) \leq C_2(\mathcal{O}_2) = \frac{1}{2}$

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Conclusion

- Preliminary study on how to define uncertainty measures on orthopairs
  - Single Orthopair: counting measure E_O
  - Collection of Orthopairs: entropy measure
- Future works
  - Better analyze the conflict case (and possibility for reconciliation)
  - Consider to measure the balance between positive and negative
  - Define a partition of orthopairs, define mutual information and use it in (rough) clustering

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